Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation, and Mathematics

PROCEEDINGS

17^{th} International Conference on Process Control 2009

Hotel Baník, Štrbské Pleso, Slovakia, June 9 – 12, 2009

ISBN 978-80-227-3081-5

http://www.kirp.chtf.stuba.sk/pc09

Editors: M. Fikar and M. Kvasnica

Králová, J., Honc, D.: Control of TISO Process - Thermostatic Bath Case Study, Editors: Fikar, M., Kvasnica, M., In *Proceedings of the 17th International Conference on Process Control '09*, Štrbské Pleso, Slovakia, 459–463, 2009.

Full paper online: http://www.kirp.chtf.stuba.sk/pc09/data/abstracts/012.html

CONTROL OF TISO PROCESS – THERMOSTATIC BATH CASE STUDY

J. Králová*, D. Honc

Faculty of Electrical Engineering and Informatics, University of Pardubice,

Studentská 95, 532 10 Pardubice
**e-mail : jaroslava.kralova@student.upce.cz*

Abstract: The paper is aimed on problematic of multivariable control. Multivariable system can be controlled by multivariable controller or we can use distributed control scheme. Control of thermal system with two inputs and one output is shown in the paper. The system is controlled by two on-off controllers, two PID controllers, split range and PID controller and by static compensator and PID controller. Control strategies are compared in the view of control quality and costs, information and knowledge required by control design and application.

Keywords: multivariable system, thermostatic bath, split range, static compensator

1 INTRODUCTION

We can define a multivariable Multi-Input Multi-Output system (MIMO) as system which has more inputs and outputs, whereas more output variables are influenced with one input.
Multivariable system can be controlled generally by multivariable controller e.g. LQ(G) controller. Such a controller requires full information about dynamic behaviour of the controlled system. Dynamic behaviour of the system can be expressed by dynamic linear mathematical model. To get reliable mathematical model of MIMO system can be difficult or even impossible (Dušek 2008).
Another possibility how to control multivariable system is to use distributed control. The controlled system is divided to several Single-Input Single-Output (SISO) subsystems with single control loops. The basic question is how to create pairs between manipulated inputs and controlled outputs. The problem is generally unsolvable without considering additional conditions in the case of different number of inputs and outputs.
If we have smaller number of inputs than outputs it is not possible to get zero steady state control error on all output variables. The solution is to specify request on degree of proximity to the set-point. This can be solved as an optimization problem dependent – solution and result depend on a criterion formulation.
If the system has more inputs than outputs the situation is more positive. This case is more interesting from practical point of view because we can get set-point with infinitely combinations of inputs. This admits to formulate additional control requirements (e.g. cost minimization). This case also leads to an optimization problem.
Couplings between the subsystems cause interaction between individual controllers, complicate controllers setting and decline of control quality (these couplings act as disturbances). Interaction can be suppressed with compensators or decouplers.
Different control strategies for system with one controlled variable and two manipulated variables (TISO) are demonstrated on practical example of thermostatic bath control. Two on-off controllers, two PID controllers, split-range and static compensator are described, designed, applied and compared.

2 CONTROLLED SYSTEM

Imperfect insulated basin filled with water C is placed in environment with temperature T_0. Electrical heating element A and coil B (pipe with flowing water) are dipped in the water. Measurement cell (element) D is also dipped in the water. Defined system has four input variables – environment temperature T_0, heating power E, temperature of cooling water T_{B0} and cooling water flow rate Q.

Output variable is temperature of water T_C or temperature of measurement cell T_D.

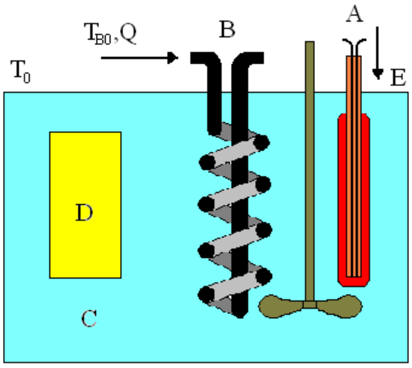

Fig. 1 Scheme of thermostatic bath

Mathematical model (Dušek 2007) can be derived under above stated assumptions, based on of thermal balance of heating element (1), coil (2), water in thermostatic bath (3) and dipped measurement cell (4). This model can be rewritten to the standard state-space form (5).

$$E = \alpha_A S_A (T_A - T_C) + m_A c_A \frac{dT_A}{dt} \qquad (1)$$

$$\mathbf{Q} c_B T_{B0} + \alpha_B S_B (T_C - T_B) = \mathbf{Q} c_B T_B + m_B c_B \frac{dT_B}{dt} \qquad (2)$$

$$\alpha_A S_A (T_A - T_C) + \alpha_D S_D (T_D - T_C) = \\ = \alpha_B S_B (T_C - T_B) + \alpha_C S_C (T_C - T_B) + m_C c_C \frac{dT_C}{dt} \qquad (3)$$

$$0 = \alpha_D S_D (T_D - T_C) + m_D c_D \frac{dT_D}{dt} \qquad (4)$$

$$\begin{bmatrix} \frac{dT_A}{dt} \\ \frac{dT_B}{dt} \\ \frac{dT_C}{dt} \\ \frac{dT_D}{dt} \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} T_A \\ T_B \\ T_C \\ T_D \end{bmatrix} + \begin{bmatrix} \frac{1}{m_A c_A} & 0 & 0 \\ 0 & \frac{\mathbf{Q}}{m_B} & 0 \\ 0 & 0 & \frac{\alpha_C S_C}{m_C c_C} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} E \\ T_{B0} \\ T_0 \end{bmatrix} \qquad (5)$$

$$\mathbf{A} = \begin{bmatrix} -\frac{\alpha_A S_A}{m_A c_A} & 0 & \frac{\alpha_A S_A}{m_A c_A} & 0 \\ 0 & -\frac{\alpha_B S_B}{m_B c_B} - \frac{\mathbf{Q}}{m_B} & \frac{\alpha_B S_B}{m_B c_B} & 0 \\ \frac{\alpha_A S_A}{m_C c_C} & \frac{\alpha_B S_B}{m_C c_C} & -\frac{\alpha_A S_A + \alpha_B S_B + \alpha_C S_C + \alpha_D S_D}{m_C c_C} & \frac{\alpha_D S_D}{m_C c_C} \\ 0 & 0 & \frac{\alpha_D S_D}{m_D c_D} & -\frac{\alpha_D S_D}{m_D c_D} \end{bmatrix}$$

where T_x are characteristic temperatures (state variable), m_x are masses, c_x are specific thermal capacities, S_x are areas for heating transfer, α_x are heat transfer coefficients between adjacent capacities, index x substitutes individual capacities A, B, C and D.

Integral part of the process properties is information about the constraints.
Parameters of model are given in Table 1, range of input variables and working point are given in Table 2 and steady state in working point in Table 3.

Tab. 1 Model parameters

Par.	**Dimension**	**A heating**	**B cooling**	**C water**	**D element**
m_x	kg	0.3	0.15669	4.0	8.93
c_x	J.kg^{-1}.K^{-1}	452	4180	4180	383
S_x	m^2	0.0095	0.065	0.24	0.06
α_x	J.m^{-2}.s^{-1}.K^{-1}	750	500	5	500

Tab. 2 Input variables – range and working point

Var.	***E* [W]**	***Q* [kg.s^{-1}]**	**T_{B0} [°C]**	**T_0 [°C]**
u_{max}	1000	0.5/60	20	25
u_0	250	0.5/60	15	25
u_{min}	0	0.5/60	5	25

Tab. 3 Steady state temperatures in working point

T_A [°C]	64.63
T_B [°C]	22.02
T_C [°C]	29.54
T_D [°C]=y_0	29.54

3 CONTROL DESIGN AND EXPERIMENTS

Following control demands and conditions are kept for all experiments. Temperature of the measurement cell T_D is selected as a controlled variable. Heating power E and temperature of cooling water T_{B0} are manipulated variables. Remaining variables are considered as disturbances. Under assumption that flow rate of the cooling water is constant, thermostatic bath is a linear system.
Control conditions are following:

- control starts from steady state - see Tab. 3
- set-point is changed stepwise in time 20 minutes from value 29.54 to value 50 and in time 80 minutes set-point returns back to 29.54
- experiment lasts 140 minutes
- sample time is 20 seconds

3.1 Two on-off controllers

On-off controller is the simplest control strategy. It is based on controller switching on and off if the set-point is met. The control can be express by following mathematical equations

$$\begin{aligned} & e(t) = w(t) - y(t) \\ & if \quad e > 0 \quad then \quad u_1 = u_{1,\max} \quad u_2 = u_{2,\max} \\ & \qquad else \quad u_1 = u_{1,\min} \quad u_2 = u_{2,\min} \end{aligned} \qquad (6)$$

Control response is drawn in Figure 2.

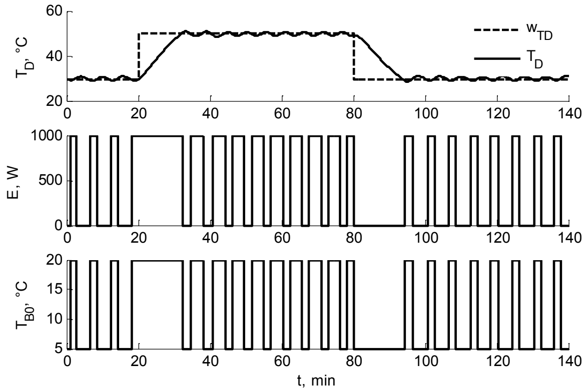

Fig. 2 Control with two on-off controllers

3.2 Two PID controllers

Equation of discrete-time PID controller is used in following form

$$u(k) = q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) + u(k-1) \quad (7)$$

where q_0, q_1 a q_2 are constants, which are calculated from continuous-time PID controller parameters according to following formulas

$$\begin{aligned} q_0 &= r_0 \left(1 + \frac{T_s}{2T_i} + \frac{T_d}{T_s} \right) \\ q_1 &= -r_0 \left(1 - \frac{T_s}{2T_i} + \frac{2T_d}{T_s} \right) \\ q_2 &= \frac{r_0 \cdot T_d}{T_s} \end{aligned} \quad (8)$$

where r_0 is controller gain, T_i is integral time constant, T_d is derivate time constant and T_s is sample time. Parameters of continuous-time PID controllers were tuned by trial-error method so control response is close to aperiodic.

Tab. 4 Parameters of PID controllers

Manipulated variable	r_0	T_i	T_d
Temperature of cooling water T_{B0}	4	4600	0
Heating power E	156	4800	0

Control response is depicted in Figure 3.

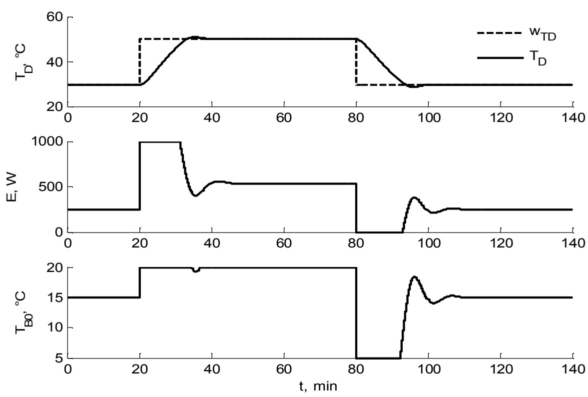

Fig. 3 Control with two PID controllers

3.3 Split-range control

Split-range strategy is often used in situations where one or more control variables should be used, depending on the operating scenario (Äström 1995). There are several reasons for splitting the signals for example dividing output of one controller into two or more signals that are applied to different control actuators.

In our case split-range is realized according to Figure 4.

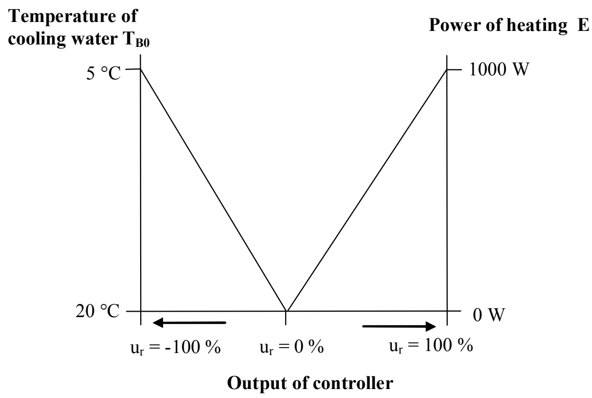

Fig. 4 Split-range scheme

We suppose that the output from controller is between -100 % and +100 %. If the controller output is negative the system is only cooled and the heating is off. Similarly if the controller output is positive cooling is on its minimal value and the heating is active. Manipulated variables are linearly interpolated according to Figure 4. If the controller output u_r = -100 % then the heating power E = 0 W and the temperature of cooling water is T_{B0} = 5 °C. If u_r = 0 % then the temperature of cooling water T_{B0} = 20 °C and the heating power E = 0 W. If u_r = 100 % then T_{B0} = 20 °C and E = 1000 W.

If the controller output is negative then the manipulated variables are

$$\begin{aligned} E &= 0 = E_{\min} \\ T_{B0} &= T_{B0,\max} + u_r \cdot \frac{(T_{B0,\max} - T_{B0,\min})}{100} \end{aligned} \quad (9)$$

where u_r is controller output, $T_{B0,max}$ is maximal cooling water temperature and $T_{B0,min}$ is minimal cooling water temperature.

If the controller output is positive then the manipulated variables are

$$\begin{aligned} E &= E_{\min} + u_r \cdot \frac{(E_{\max} - E_{\min})}{100} = u_r \cdot \frac{(E_{\max} - E_{\min})}{100} \\ T_{B0} &= 20 = T_{B0\max} \end{aligned} \quad (10)$$

where u_r is controller output, E_{max} is maximal heating power, E_{min} is minimal heating power.

Discrete-time PID controller is used with Split-range. Parameters of the controller are calculated from continuous-time PI controller parameters tuned with trial-error method to get aperiodic control responses. The controller gain is r_0 = 24 and integral time constant is T_i = 1250 s. Control response is depicted in Figure 5.

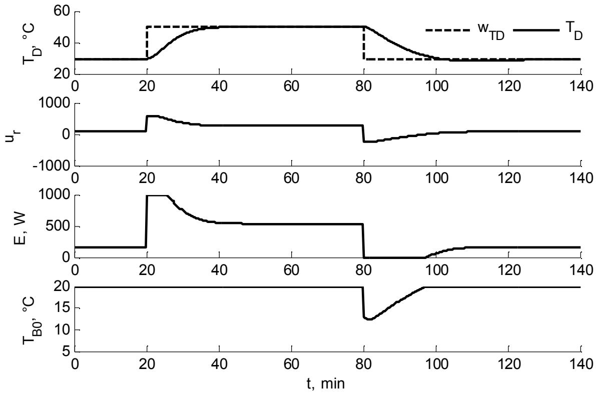

Fig. 5 Control with Split-range

3.4 Control with static compensator

The basic idea is that for system with two inputs and one output we can reach set-point with infinity combinations of manipulated variables. Another control request may be specified. For example we can require that inputs of the system (manipulated variables) are as close as possible to desired inputs u_{w1} a u_{w2} (Honc 2008). This is an optimization problem – minimization under constrains existing. If we choose quadratic criterion with weighting coefficients m_1 and m_2, we can describe the problem as

$$\min_{u_1,u_2} J(u_1,u_2) = m_1(u_1-u_{w1})^2 + m_2(u_2-u_{w2})^2,$$
$$\text{constrain } y = Z_1u_1 + Z_2u_2 \tag{11}$$

Real inputs are approaching to their desired values depending on the weighting coefficients m_1 and m_2. Minimization with bounded extreme can be solved by the help of Lagrange's multipliers with criterion in form

$$\min_{u_1,u_2,\lambda} J(u_1,u_2,\lambda) = m_1(u_1-u_{w1})^2 + m_2(u_2-u_{w2})^2 + \lambda(Z_1u_1+Z_2u_2-y) \tag{12}$$

If we want to calculate minimum of the function (12), we have to derive partial derivations, put them equal to zero and then to solve arising equation set. Solution of this equation set is

$$\begin{bmatrix}u_1\\u_2\\\lambda\end{bmatrix} = \frac{1}{m_1Z_2^2+m_2Z_1^2}\begin{bmatrix}m_1Z_2^2 & -m_2Z_1Z_2 & m_2Z_1\\ -m_1Z_1Z_2 & m_2Z_1^2 & m_1Z_2\\ 2m_2Z_1 & 2m_1Z_2 & -2m_1m_2\end{bmatrix}\cdot\begin{bmatrix}u_{w1}\\u_{w2}\\y\end{bmatrix} \tag{13}$$

Only inputs are required for control

$$\begin{bmatrix}u_1\\u_2\end{bmatrix} = \frac{1}{m_1Z_2^2+m_2Z_1^2}\begin{bmatrix}m_1Z_2^2 & -m_2Z_1Z_2 & m_2Z_1\\ -m_1Z_1Z_2 & m_2Z_1^2 & m_1Z_2\end{bmatrix}\cdot\begin{bmatrix}u_{w1}\\u_{w2}\\y\end{bmatrix} \tag{14}$$

From equation (14) we can calculate inputs of the system, which are close to desired values and guarantee output y in the steady state. If we require unit steady state gain of the compensated system, process output y must be equal to controller output u_r.

$$\begin{bmatrix}u_1\\u_2\end{bmatrix} = \underbrace{\frac{1}{m_1Z_2^2+m_2Z_1^2}\begin{bmatrix}m_1Z_2^2 & -m_2Z_1Z_2\\ -m_1Z_1Z_2 & m_2Z_1^2\end{bmatrix}}_{\mathbf{R}}\cdot\begin{bmatrix}u_{w1}\\u_{w2}\end{bmatrix} + \underbrace{\frac{1}{m_1Z_2^2+m_2Z_1^2}\begin{bmatrix}m_2Z_1\\m_1Z_2\end{bmatrix}}_{\mathbf{S}}\cdot u_r = \mathbf{R}\cdot\begin{bmatrix}u_{w1}\\u_{w2}\end{bmatrix} + \mathbf{S}\cdot u_r \tag{15}$$

Static compensator modifies system with two inputs and one output to a system with one input and one output. Static compensator splits controller output into two control actions optimally according to cost function and weighting coefficients.

The temperature of cooling water T_{B0} represents input u_1 and the heating power E represents input u_2. The gains of the system are $Z_1 = 0.0011$, $Z_2 = 6.3784.10^{-5}$.

We need to choose values of desired inputs: u_{w1} = 20 °C and u_{w2} = 0 W.

The weighting coefficients are chosen as input costs. We want to have such a combination of inputs so that we get minimal expenses on heating and cooling.

The weighting coefficients are calculated according to following equations

$$m_1 = \left(c_j\cdot\frac{Q\cdot c_{H_2O}}{\varepsilon}\right)^2 \tag{16}$$
$$m_2 = c_j^2$$

where Q is the cooling water flow rate, c_{H_2O} is the thermal capacity of water, ε is the cooling efficiency (ε = 40 %), c_j is price of 1 J energy in Czech crowns (c_j = 4/3600/1000).

Control response with static compensator and discrete-time PID controller is drawn in Figure 6. Parameters of the controller are calculated from continuous-time PI controller parameters tuned with trial-error method to get aperiodic control responses. The gain of controller is r_0 = 0.007 and integral time constant is T_i = 2500 s.

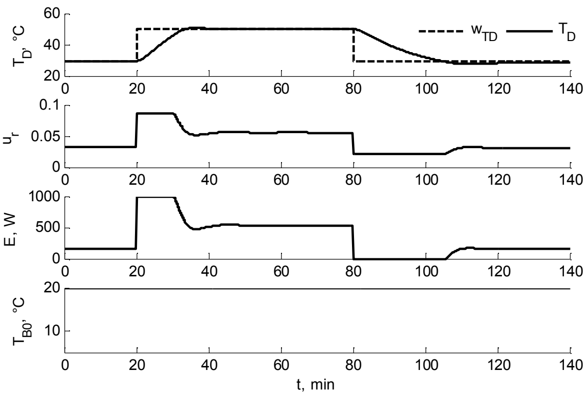

Fig. 6 Control with static compensator

4 CONTROL PERFORMANCE MEASURES

Three measures are computed to compare discussed control methods from the view of control quality and heating and cooling costs.

4.1 Control quality measure K

This measure is defined as a square root from mean quadratic control error

$$K = \sqrt{\frac{1}{N}\sum_{i=1}^{N} e^2(i)} \tag{17}$$

where N is a number of samples in the experiment.

4.2 Heating cost N_h

Heating cost is calculated directly from the price of electric energy

$$N_h = c_j \cdot T_s \cdot \sum_{i=1}^{N} E(i) \tag{18}$$

where T_s is a sample time.

4.3 Cooling cost N_c

Cooling cost is calculated in following way. To cool down the water the same amount of energy is necessary as to heat it up plus energy to respect lower efficiency of the cooling compared to the heating.

$$N_c = c_j \cdot \frac{Q \cdot c_{H_2O}}{\varepsilon} \cdot T_s \cdot \sum_{i=1}^{N} \left(T_{B0\max} - T_{B0}(i)\right) \tag{19}$$

where $T_{B0\max}$ is maximal cooling water temperature and $T_{B0}(i)$ is temperature of the cooling water.

4.4 Evaluated performance measures

Above stated measures are computed for all experiments. We can compare the control quality and costs of individual control methods according to Table 5.

Tab. 5 Control performance measures

Experiment	K	N_h (Kč)	N_c (Kč)	N_h+N_c (Kč)
two on-off controllers	5.21	4.33	6.53	10.86
two PID controllers	5.45	3.62	3.06	6.68
Split-range	5.78	3.10	0.42	3.52
Static compensator	6.07	3.05	0	3.05

5 CONCLUSION

The paper is aimed on temperature control in thermostatic bath as a system with two inputs and one output. The system is controlled by

a) two on-off controllers
b) two PID controllers
c) split-range and PID controller and
d) static compensator and PID controller.

For all experiments performance measures are calculated. These measures characterize control performance in the view of control quality and costs connected with heating and cooling. The values of these criteria are recorded in Table 5.

If we compare control quality measure, we can state that the best control performance is with two on-off controllers for this case. Coincidentally the heating was switched on and the cooling was switched off even before the set-point change in time 20 minutes. This situation is advantageous in term of control quality. An oscillation of controlled variable around the set-point value because of switching minimal and maximal control action did not appear on control quality (generally on-off controller should be worse than continuous controller with zero control error). The control with two on-off controllers is the most expensive control method.

Temperature control with static compensator and PID controller and split-range and PID controller are the best methods from the control costs point of view.

Two PID controllers do not fulfil condition of optimal cost in the steady state - manipulated variables freeze after the control error is zero.

ACKNOWLEDGMENTS

The work has been supported by program of Czech Republic MSM 0021627505. This support is very gratefully acknowledged.

REFERENCES

ÄSTRÖM, K.; HÄGGLUND, T. *PID Controllers: Theory, Design, and Tuning 2nd edition*. North Carolina: The International Society for Measurement and Control, 1995. 343 pp. ISBN 1-55617-516-7.

DUŠEK, F.; HONC, D. Transformace soustav s různým počtem vstupů a výstupů pro decentralizované řízení. *Automatizace*, 2008, č. 7-8, s. 458 - 462. ISSN 0005-125X.

DUŠEK, F.; HONC, D. Návrh a simulace řízení nesymetrického systému. *Automatizace*, 2007, č. 10, s. 636 - 642.

HONC, D.; DUŠEK, F. Řízení soustavy se dvěmi vstupy a jedním výstupem. *Perner's contacts*, 2008, č. V, s. 107-112. ISSN 1801-674X.