Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation, and Mathematics

PROCEEDINGS

17$^{\text{th}}$ International Conference on Process Control 2009

Hotel Baník, Štrbské Pleso, Slovakia, June 9 – 12, 2009

ISBN 978-80-227-3081-5

http://www.kirp.chtf.stuba.sk/pc09

Editors: M. Fikar and M. Kvasnica

Švejda, M.: Application of Output Tracking for DC Motor with Flexible Shaft, Editors: Fikar, M., Kvasnica, M., In *Proceedings of the 17th International Conference on Process Control '09*, Štrbské Pleso, Slovakia, 464–470, 2009.

Full paper online: http://www.kirp.chtf.stuba.sk/pc09/data/abstracts/071.html

APPLICATION OF OUTPUT TRACKING FOR DC MOTOR WITH FLEXIBLE SHAFT

Martin Švejda *

* *University of West Bohemia in Pilsen, Faculty of Applied Sciences, Department of Cybernetics, Univerzitni 22, 306 14 Pilsen*
tel : +420 377 632 502 and e-mail : msvejda@kky.zcu.cz

Abstract: The paper deals with the application of the output tracking method for the controlled system represented by the DC motor with a flexible shaft. The proposed output tracking method uses the known results of the state coordinate transformation and tracking convergence for an integrator chain. The control law is designed in such a way that ensures the tracking error to converge to zero exponentially. The robustness of the proposed method is discussed. At last, the simulation results are presented for the controlled system without uncertainties and for the perturbed controlled system with one unknown parameter of the flexible shaft.

Keywords: output tracking, DC motor with a flexible shaft, normal form, integration chain, robustness

1. INTRODUCTION

Research of the precise motion control is very important for many industrial applications, for example the motion control of the robots, the manipulators and the electronic cams. The problem of the output tracking involves a design of the control law such that the output of the controlled system follows the desired trajectory. This paper demonstrates the output tracking of the linear SISO controlled system represented by the DC motor with a flexible shaft. The main aim of the output tracking of the DC motor with a flexible shaft is to ensure that the angular position of the flexible shaft will follow the desired angular position.

The proposed output tracking controller is based on the transformation of the controlled system to the so-called normal form. This transformation is the application of the partial feedback linearization of the nonlinear control systems from Isidori (1989) to the linear SISO systems. Consequently, the standard tracking convergence for the integrator chain presented by Getz (1995) can be used. It is shown that the proposed output tracking controller ensures the convergence angular position of the flexible shaft to desired angular position exponentially. However, it is difficult to determine accurate mathematical description for the real physical system represented by the DC motor with a flexible shaft. Therefore, the robustness of the proposed output tracking controller is discussed regarding uncertainties consisted of the perturbed parameters of the controlled system as well as external disturbances.

2. OUTPUT TRACKING FOR LINEAR SISO SYSTEM

Consider the following linear minimum-phase SISO controlled system without uncertainties

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{1\times n}$

By using the state coordinate transformations, the controlled system (1) can be put to its so-called *normal form*. See Isidori (1989).

A relative degree of the controlled system (1) is defined to be the least positive integer ρ such that

$$CA^{\rho-1}B \neq 0 \tag{2}$$

This implies that if we differentiate y with respect to t, the input u appears for the first time at the ρ-th derivative of the output y. We use this fact in the following definition.

Define partial coordinate transformation

$$\xi_k = y^{(k-1)} = T_k x \tag{3}$$

where $T_k = CA^{k-1}$ and $k = 1 \dots \rho$

To complete partial coordinate transformation (3) we have to choose remaining $n - \rho$ functions

$$\eta_i = T_{\rho+i} x \tag{4}$$

where $T_{\rho+i} \in \mathbb{R}^{1\times n}$ and $i = 1 \dots n - \rho$

It is shown by Isidori (1989), that for the controlled system (1) always exist vectors $T_{\rho+i}$ satisfying

$$T_{\rho+i}B = 0 \tag{5}$$

and the transformation matrix

$$T = \begin{bmatrix} T_1 \\ \vdots \\ T_\rho \\ T_{\rho+1} \\ \vdots \\ T_n \end{bmatrix} \tag{6}$$

is regular ($\det T \neq 0$).

It can be shown that, coordinate transformation

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = Tx \tag{7}$$

where $\xi = [\xi_1 \dots \xi_\rho]^T$ and $\eta = [\eta_1 \dots \eta_{n-\rho}]^T$

brings the controlled system (1) to so-called *normal form*

$$E_{ext} : \begin{cases} \dot{\xi}_k = \xi_{k+1} & k = 1 \dots \rho - 1 \\ \dot{\xi}_\rho = \mathbf{b} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \mathbf{a}u \end{cases} \tag{8}$$
$$E_{int} : \left\{ \dot{\eta} = \mathbf{q} \begin{bmatrix} \xi \\ \eta \end{bmatrix} \right.$$
$$y = \xi_1$$

where

$$\mathbf{b} = CA^\rho T^{-1}$$
$$\mathbf{a} = CA^{\rho-1}B$$
$$\mathbf{q} = \left[T_{\rho+1}AT^{-1}, \dots, T_n AT^{-1}\right]^T$$

We can see that the controlled system (8) consists of two parts, the external dynamics E_{ext} and the internal dynamics E_{int}.

2.1 *Zero dynamics*

Notice, that the internal dynamics E_{int} of the controlled system (8) is related to the so-called zero dynamics of the controlled system (1). We define the zero dynamic as follows.

Assume, that we want to ensure zero output $y = 0$ of the controlled system (8) for all time. It means (recall $y = \xi_1$) that for all time $\xi = \mathbf{0}$ and the input u must necessary be the solution of the equation

$$0 = \dot{\xi}_\rho = \mathbf{b} \begin{bmatrix} 0 \\ \eta \end{bmatrix} + \mathbf{a}u$$
$$u = -a^{-1}b \begin{bmatrix} 0 \\ \eta \end{bmatrix}$$

where η satisfies the differential equation

$$\dot{\eta} = \mathbf{q} \begin{bmatrix} 0 \\ \eta \end{bmatrix} \tag{9}$$

for arbitrary initial condition η_0.

Consequently, the dynamics given by the equation (9) corresponds to the dynamics describing the "internal" behavior of the controlled system (1) when the initial conditions and the input are chosen to zero the output. We call this dynamics the *system's zero dynamics*. It can be shown that the eigenvalues of the zero dynamics represented by equation (9) corresponds to the zeros of the controlled system (1).

Now, we consider the controlled system (1) in its normal form (8) and define the control law

$$u = \mathbf{a}^{-1} \left(-\mathbf{b} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + w \right) \tag{10}$$

Substituting (10) into (8) results in

$$E_{ext} : \begin{cases} \dot{\xi}_k = \xi_{k+1} & k = 1 \dots \rho - 1 \\ \dot{\xi}_\rho = w \end{cases} \tag{11}$$
$$E_{int} : \left\{ \dot{\eta} = \mathbf{q} \begin{bmatrix} \xi \\ \eta \end{bmatrix} \right.$$
$$y = \xi_1$$

We can see that there is only the integration chain between the input w and the output y. If we suppose the minimum-phase controlled system (1), the zero dynamic (9) is stable. It implies that the internal dynamics E_{int} of the controlled system (11) is bounded.

2.2 *Tracking convergence for integration chain*

According to Getz (1995), to ensure the output y of the controlled system (11) follows the desired trajectory y_d the standard result of the linear control theory "*tracking convergence for integration chain*" can be used.

We define a polynomial

$$s^{\rho} + \sum_{j=1}^{\rho} \beta_j s^{j-1} = 0 \tag{12}$$

where the coefficients $\beta_j \in \mathbb{R}$ are chosen in order that all the roots of the polynomial have strictly negative real part. Then the control law

$$w = y^{(\rho)} - \sum_{j=1}^{\rho} \beta_j \left(\xi_j - y_d^{(j-1)} \right) \tag{13}$$

where $y_d \in \mathcal{C}^{\rho}$ is the desired trajectory,

causes y converges to y_d exponentially.

We can prove this fact by analyzing the tracking error coordinates

$$e_k = \xi_k - y_d^{(k-1)} \tag{14}$$

It is clear that the external dynamics E_{ext} of the controlled system (11) with the control law (13) for the error coordinates (14) takes a form

$$\begin{aligned} \dot{e}_k &= e_{k+1} \qquad k = 1 \dots \rho - 1 \\ \dot{e}_{\rho} &= - \sum_{j=1}^{\rho} \beta_j e_j \end{aligned} \tag{15}$$

The error system (15) is asymptotically stable and speed of the convergence to zero depends on the parameters β_j.

3. ROBUSTNESS OF PROPOSED METHOD

Now, we suppose that the controlled system (1) is affected by some uncertainties. The uncertain controlled system can be written

$$\begin{aligned} x &= Ax + Bu + \Delta Ax + \Delta Bu \\ y &= Cx \end{aligned} \tag{16}$$

where ΔA and ΔB are unknown matrices with appropriate dimension.

We define the characteristic index of uncertainties ΔAx to be the least positive integer σ such that

$$CA^{\sigma-1}\Delta A \neq 0 \tag{17}$$

and the characteristic index of uncertainties ΔBu to be the least positive integer ν such that

$$CA^{\nu-1}\Delta B \neq 0 \tag{18}$$

We suppose that the unknown matrices hold following properties

$$\nu \geq \rho \geq \sigma > 1 \tag{19}$$

where ρ is the relative degree of the uncertain controlled system (16). See (2).

It can be shown that uncertain controlled system (16), through the coordinate transformation (7), takes a form

$$E_{ext} : \begin{cases} \dot{\xi}_k = \xi_{k+1} \\ \dot{\xi}_{\sigma+j} = \xi_{\sigma+j+1} + \delta_{\sigma+j-1} \begin{bmatrix} \xi \\ \eta \end{bmatrix} \\ \dot{\xi}_{\rho} = \mathbf{b} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \mathbf{a}u + \mathbf{\Delta b} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \mathbf{\Delta a}u \end{cases} \tag{20}$$

$$E_{int} : \left\{ \dot{\eta} = \mathbf{q} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \mathbf{\Delta q} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \mathbf{\Delta r}u \right.$$

$$y = \xi_1$$

where $k = 1 \dots \sigma - 1$, $j = 0 \dots \rho - \sigma - 1$

$$\begin{aligned} \delta_{\sigma+\mathbf{j}-\mathbf{1}} &= CA^{\sigma+j-1}\Delta AT^{-1} \\ \mathbf{b} &= CA^{\rho}T^{-1} \\ \mathbf{\Delta b} &= CA^{\rho-1}\Delta AT^{-1} \\ \mathbf{a} &= CA^{\rho-1}B \\ \mathbf{\Delta a} &= CA^{\rho-1}\Delta B \\ \mathbf{q} &= \left[T_{\rho+1}AT^{-1}, \dots, T_n AT^{-1}\right]^T \\ \mathbf{\Delta q} &= \left[T_{\rho+1}\Delta AT^{-1}, \dots, T_n \Delta AT^{-1}\right]^T \\ \mathbf{\Delta r} &= \left[T_{\rho+1}\Delta B, \dots, T_n \Delta B\right]^T \end{aligned}$$

Substituting the control law (10) to the controlled system (20) we get

$$E_{ext} : \begin{cases} \dot{\xi}_k = \xi_{k+1} \\ \dot{\xi}_{\sigma+j} = \xi_{\sigma+j+1} + \delta_{\sigma+j-1} \begin{bmatrix} \xi \\ \eta \end{bmatrix} \\ \dot{\xi}_{\rho} = \left[\mathbf{\Delta b} - \mathbf{\Delta a}\mathbf{a}^{-1}\mathbf{b}\right] \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \left[1 + \mathbf{\Delta a}\mathbf{a}^{-1}\right] w \end{cases} \tag{21}$$

$$E_{int} : \left\{ \dot{\eta} = \left[\mathbf{q} + \mathbf{\Delta q} - \mathbf{\Delta r}\mathbf{a}^{-1}\mathbf{b}\right] \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \mathbf{\Delta r}\mathbf{a}^{-1}w \right.$$

$$y = \xi_1$$

Obviously, the uncertain controlled system (21) does not contain the integration chain from the input w to the output y as in (11). This implies that using the control law (13) does not cause the output y to converge to y_d exponentially.

4. DC MOTOR WITH A FLEXIBLE SHAFT

Consider the following model of the DC motor with a flexible shaft. See Fig. 1. where **PI** denotes

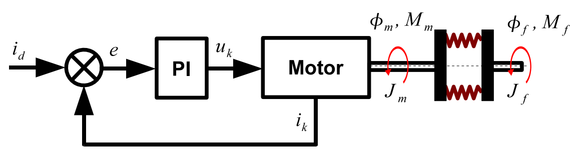

Fig. 1. Model of the DC motor with a flexible shaft

the PI current controller of the motor, u_k, i_k are the voltage and current of the armature coil of the motor, i_d is the current setpoint, ϕ_m, ω_m is the angle and angular velocity of the motor's shaft, ϕ_f, ω_f is the angle and angular velocity of the flexible shaft, J_m is the moment of inertia of the motor's shaft and J_f is the moment of inertia of the flexible shaft.

M_m is the electromechanical torque of the motor given by

$$M_m = K_t i_k \tag{22}$$

where K_t is the torque constant.

M_f is the torque of the flexible shaft and we suppose that M_f depends on the difference between the motor angle ϕ_m and the flexible shaft angle ϕ_f

$$M_f = k\left(\phi_m - \phi_f\right) \tag{23}$$

where k is the spring constant.

We assume the following description of the electric part of the motor

$$\frac{di_k}{dt} = -\frac{R}{L}i_k - \frac{K_e}{L}\omega_m + \frac{1}{L}u_k \tag{24}$$

where L and R is the inductance and resistance of the armature coil and K_e is the speed constant.

The PI current controller define as

$$u_k = \alpha\left(e + \frac{1}{T_i}\int e\,dt\right) \tag{25}$$

where $e = i_d - i_k$ is the control deviation and T_i is the integral time constant.

State space model of the PI current controller with the new state variable r can be written

$$\begin{aligned}\frac{dr}{dt} &= e \\ u_k &= \frac{\alpha}{T_i}r + \alpha e\end{aligned} \tag{26}$$

From torque balances of the motor's shaft and the flexible shaft we get

$$\begin{aligned}M_e &= J_m\frac{d\omega_m}{dt} + B_m\omega_m + M_f \\ M_f &= J_f\frac{d\omega_f}{dt} + B_f\omega_f\end{aligned} \tag{27}$$

where B_m and B_f are the viscous friction coefficients of the motor's shaft and the flexible shaft.

Substituting (23) and (22) into (27) results in the mechanical description of the controlled system

$$\begin{aligned}\frac{d\omega_m}{dt} &= \frac{K_t}{J_m}i_k - \frac{B_m}{J_m}\omega_m - \frac{k}{J_m}\phi_m + \frac{k}{J_m}\phi_f \\ \frac{d\omega_f}{dt} &= -\frac{B_f}{J_f}\omega_f + \frac{k}{J_f}\phi_m - \frac{k}{J_f}\phi_f\end{aligned} \tag{28}$$

Considering the electric part of the motor (24), the state space model of the PI current controller (26), the electromechanical description (28) and the relations $\omega_m = \frac{d\phi_m}{dt}$ and $\omega_f = \frac{d\phi_f}{dt}$ we get the complete state space model of the DC motor with a flexible shaft

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \tag{29}$$

where

$$x = \left[i_k\ r\ \phi_m\ \omega_m\ \phi_f\ \omega_f\right]^T$$
$$u = i_d$$
$$y = \phi_f$$

$$A = \begin{bmatrix} -\left(\frac{R}{L} + \frac{\alpha}{L}\right) & \frac{\alpha}{LTi} & 0 & -\frac{K_e}{L} & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{K_t}{J_m} & 0 & -\frac{k}{J_m} & -\frac{B_m}{J_m} & \frac{k}{J_m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{k}{J_f} & 0 & -\frac{k}{J_f} & -\frac{B_f}{J_f} \end{bmatrix}$$

$$B = \left[\frac{\alpha}{L}\ 1\ 0\ 0\ 0\ 0\right]^T \qquad C = \left[0\ 0\ 0\ 0\ 1\ 0\right]$$

The transfer function of the controlled system (29) is defined by the equation

$$F(p) = \frac{C\,\mathrm{adj}\left(p\mathbf{I} - A\right)B}{\det\left(p\mathbf{I} - A\right)} \tag{30}$$

where roots of the polynomial $C\,\mathrm{adj}\left(p\mathbf{I} - A\right)B$ are zeros of the controlled system. Thus, it can be shown that the controlled system has only one stable zero $z = -\frac{1}{T_i}$ given by the integral time constant of the PI current controller. This implies that the relative degree of the controlled system is $\rho = 5$.

Notice, that according to (4)-(6) the vector T_6 can be chosen as

$$T_6 = \left[1\ -\frac{alpha}{L}\ 0\ 0\ 0\ 0\right] \tag{31}$$

5. SIMULATION RESULTS

Consider the brushed DC motor *A-max 32* supplied by *Maxon motor* with following parameters: $R = 7.17\ \Omega$, $L = 9.53 \cdot 10^{-4}\ H$, $K_t = 4.6 \cdot 10^{-4}\ \frac{Nm}{A}$, $K_e = 0.29\ Vs$, $J_m = 4.4 \cdot 10^{-5}\ kg\,m^2$, $B_m = 7.05 \cdot 10^{-5}\ Nm\,s$ For more details see Maxon.

The flexible shaft parameters:
$k = 0.01\ \frac{N}{m}$, $J_f = 2 \cdot 10^{-5}\ kg\,m^2$, $B_f = 3 \cdot 10^{-5}\ Nm\,s$

The PI current controller parameters:
$\alpha = 11$, $T_i = 5 \cdot 10^{-4}$

We assume the desired angular position of the flexible shaft in the following form

$$y_d = 3\sin\left(11t\right) + 2\cos\left(8t + 0.5\right) \tag{32}$$

The simulation model of the DC motor with a flexible shaft is designed in Matlab-Simulink

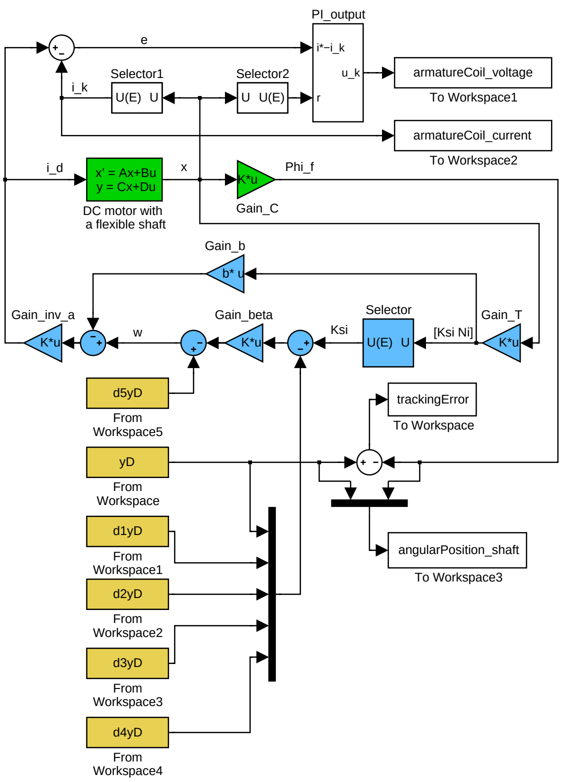

Fig. 2. Simulink model of the controlled system

and shown in Fig. 2. Green blocks correspond to the model of the DC motor with a flexible shaft, blue blocks correspond to the proposed controller and yellow blocks correspond to the generator of the desired angular position of the shaft and its appropriate derivatives. The parameters $\{\beta_1\ \beta_2\ \beta_3\ \beta_4\ \beta_5\}$ are chosen $\{55440,\ 31594,\ 7155,\ 805,\ 45\}$ which leads to the stable poles $\{-7,\ -8,\ -9,\ -10,\ -11\}$ of the error system (15).

Firstly, we suppose all parameters of the controlled system (29) perfectly known. The following figures show tracking history of the angular position of the shaft (Fig. 3), tracking error given by $error[rad] = y_d - y$ (Fig. 4), voltage (Fig. 5) and current (Fig. 6) of the armature coil of the motor.

Secondly, all parameters of the controlled system are perfectly known except the moment of inertia of the flexible shaft. Let

$$J_f = J_f^n + \Delta J_f \tag{33}$$

where J_f^n is the nominal value of the moment of inertia of the flexible shaft and ΔJ_f is the unknown perturbation. For example, let this perturbation corresponds to a load of the flexible shaft.

Thus, for the uncertain controlled system (16) with perturbation ΔJ_f holds

$$x = \begin{bmatrix} i_k & r & \phi_m & \omega_m & \phi_f & \omega_f \end{bmatrix}^T$$
$$u = i_d$$
$$y = \phi_f$$

$$A = \begin{bmatrix} -\left(\frac{R}{L}+\frac{\alpha}{L}\right) & \frac{\alpha}{LTi} & 0 & -\frac{K_e}{L} & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{K_t}{J_m} & 0 & -\frac{k}{J_m} & -\frac{B_m}{J_m} & \frac{k}{J_m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{k}{J_f^n} & 0 & -\frac{k}{J_f^n} & -\frac{B_f}{J_f^n} \end{bmatrix}$$

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{63} & 0 & a_{65} & a_{66} \end{bmatrix}$$

where

$$a_{63} = \frac{-k\Delta J_f}{J_f^n(J_f^n + \Delta J_f)} \qquad a_{65} = \frac{k\Delta J_f}{J_f^n(J_f^n + \Delta J_f)}$$
$$a_{66} = \frac{B_f\Delta J_f}{J_f^n(J_f^n + \Delta J_f)}$$

$$B = \begin{bmatrix} \frac{\alpha}{L} & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \qquad \Delta B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$
$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

It can be shown that the characteristic indexes (17) and (18) will be $\sigma = 2$, $\nu = \infty$. The following figures show tracking history of the angular position of the shaft (Fig. 7), tracking error given by $error[rad] = y_d - y$ (Fig. 8), voltage (Fig. 9) and current (Fig. 10) of the armature coil of the motor. The unknown perturbation is chosen as $\Delta J_f = 0.05 J_f^n$.

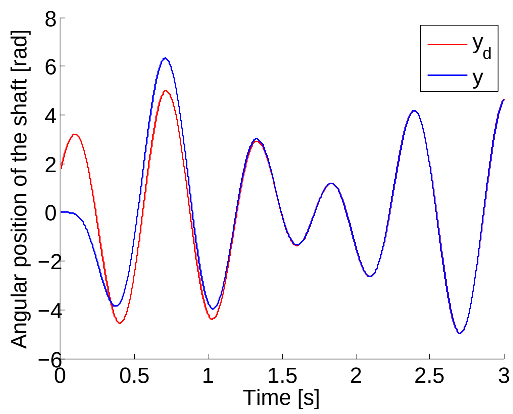

Fig. 3. Tracking history (without uncertainties)

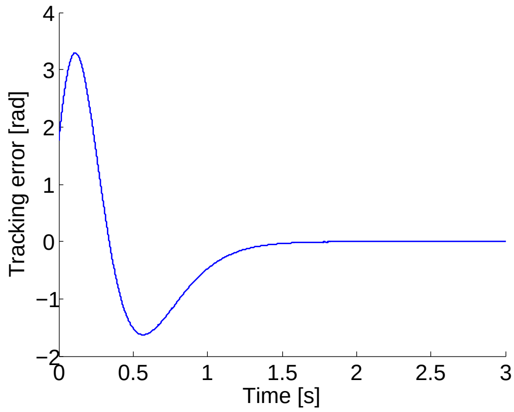

Fig. 4. Tracking error (without uncertainties)

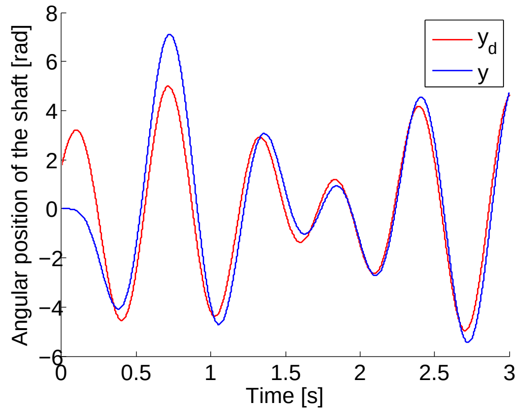

Fig. 7. Tracking history (J_f is perturbed)

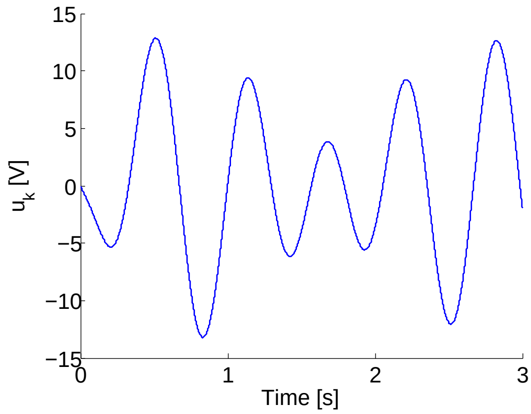

Fig. 5. Voltage of the armature coil of the motor (without uncertainties)

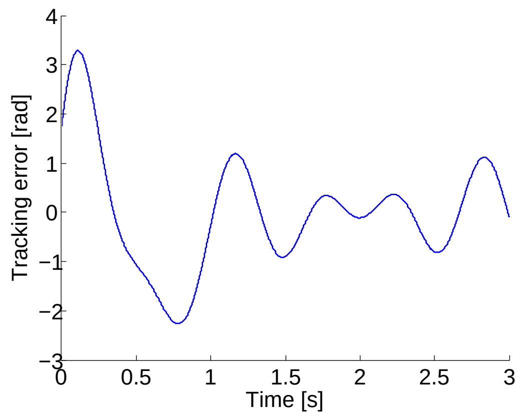

Fig. 8. Tracking error (J_f is perturbed)

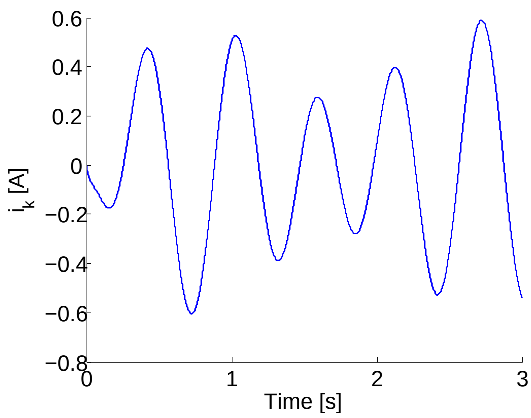

Fig. 6. Current of the armature coil of the motor (without uncertainties)

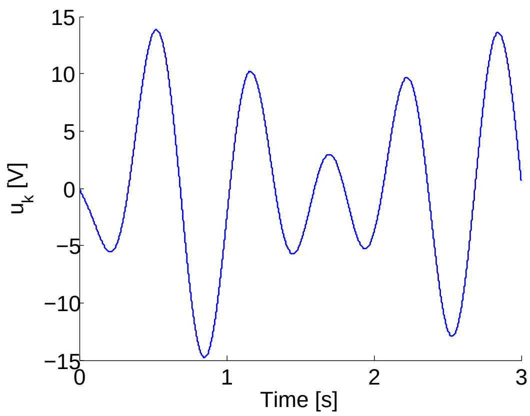

Fig. 9. Voltage of the armature coil of the motor (J_f is perturbed)

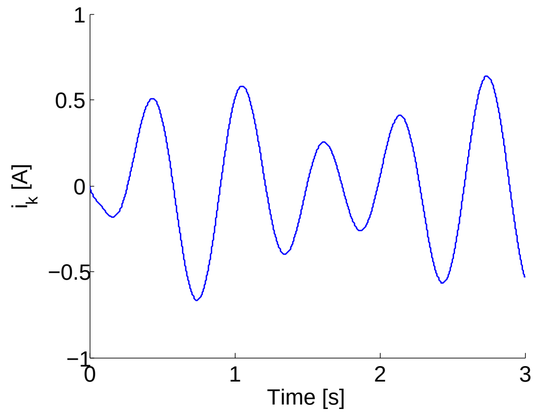

Fig. 10. Current of the armature coil of the motor (J_f is perturbed)

6. CONCLUSION

This paper presents the application of one of the output tracking methods to the electromechanical system represented by DC motor with a flexible shaft. It is shown that the proposed method ensures exponentially convergence of the tracking error only for perfectly known parameters of the controlled system. In the presence of the uncertainties actuating on the flexible shaft the proposed method fails and leads to nonzero tracking error. In order to reduce the influence of the uncertainties on the controlled system the robustness properties of the sliding mode control presented in Barbot (2002) or the robust output tracking via a modified optimal linear quadratic method in Shieh et al. (2003) can be used. But these methods require the controlled system to fulfill the so-called *matching condition*. Notice, that the controlled system (21) satisfies the matching condition only if $\sigma = \rho$. If $\sigma < \rho$ the controlled system (21) is denoted as the *mismatched uncertain system*. The robust output tracking method for mismatched uncertain systems is presented in Li et al. (1995) and Wang et al. (1998)

References

J.P. Barbot. *Sliding Mode Control in Engineering*. Marcel Dekker, New York, 2002.

N. H. Getz. *Dynamic Inversion of Nonlinear Maps with Applications to Nonlinear Control and Robotics*. PhD thesis, University of California at Berkeley, 1995.

A. Isidori. *Nonlinear Control Systems, An Introduction*. Springer-Verlag, second edition, New York, 1989.

Z. Li, T.Y. Chai, and C.B. Soh C. Wen. Robust output tracking for nonlinear uncertain system. *System and Control Letters*, 25:53–61, 1995.

Maxon. Maxon motor. https://shop.maxonmotor.com/ishop/article /article/236663.xml.

N.C. Shieh, K.Z. Liang, and C.J. Mao. Robust output tracking control of an uncertain linear system via a modified optimal linear-quadratic method. *Optimization Theory and Applications*, 117:649–659, 2003.

W.H. Wang, C.B. Soh, and T.Y. Chai. Robust output tracking for mimo nonlinear time-varying mismatched uncertain systems via hybrid control strategy. *Dynamics and Control*, 8: 191–208, 1998.