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# SOME ASPECTS OF EXPONENTIAL STABILITY FOR NETWORKED CONTROL SYSTEMS WITH RANDOM DELAYS

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Abstract: In this paper, the problem of stability for the standard form of state control, realized in a networked control system structure, is studied. To deal with the problem of stability analysis of event–time–driven modes in networked control systems the delayed–dependent exponential stability condition are proven and actualized. Based on the delay-time dependent Lyapunov-Krasovskii functional the linear matrix inequalities for stability conditions are new formulated. Since presented method can use bilinear matrix inequality techniques it is computationally enough efficient.

Keywords: Networked systems, stability analysis, time-delay systems, linear matrix inequality, state feedback.

## 1. INTRODUCTION

Recent advances in communication technology lead to an increased use of networked control. Networked control systems (NCS) are control loop closed through a shared communication network, where network between control system components is used to exchange the information and control signals. The advantage of such structure are most of all simple installation, maintenance and system volume, increased system agility. However, due to communication channel insertion, induced delay and packet dropout may seriously deteriorate the performance of the system, especially its stability.

During the previous decade, the stability problem of networked control systems with random delays has attracted a lot of attention. The main approach for stability analysis relies on Lyapunov–Krasovskii functionals and LMI approach for constructing common Lyapunov function. For the reason of network-induced delays it is often assumed that the actuator and controller are event driven, but once the large delay bound appears, system may become unstable. The usual approach ignores in the controller design stage the network delays and in the next design step there is analyze stability, performance and robustness with respect to the effects and characteristics of network delays and scheduling policy. Some progres review in this research field one can find in Gu et al. (2003), Dritsas and Tzes. (2008), and the references therein.

This paper is concerned with the problem of event–time–driven mode in networked control system. Under this mode in a critical event a switched delay system structure is occasioned, which may include an unstable subsystem. Paper actualizes, completes and extends the basic idea presented in Sun et al. (2008) in coincidence with Zhang et al. (1999) to obtain conditions for exponential stability of a such structure. Possibly time–varying delay is considered and attention is focused on linear matrix inequalities (LMIs) which have to hold to obtain control exponentially stable. Presented LMI approach is computationally efficient as it can be solved numerically (see e.g. Boyd at al. (1994)), and is based on Lyapunov–Krasovskii functional (see e.g. Kolmanovskii et al. (1999)) and the Leibniz–Newton formula to eliminate some dead-time dependent terms (Park (1999)).

## 2. PROBLEM DESCRIPTION

Through this paper the task is concerned with stability analysis of NCS, where a linear dynamic system given by the set of equations

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{2}$$

is controlled by delayed state feedback. Here $\boldsymbol{q}(t) \in I\!R^n$, $\boldsymbol{u}(t) \in I\!R^r$, and $\boldsymbol{y}(t) \in I\!R^m$ are vectors of the state, input and measurable output variables, respectively, and system matrices $\boldsymbol{A} \in I\!R^{n\times n}$, $\boldsymbol{B} \in I\!R^{n\times r}$ and $\boldsymbol{C} \in I\!R^{m\times n}$ are real matrices. Problem of the interest is to design stable NCS with the linear memoryless state feedback controller of the form

$$\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{q}(t) \tag{3}$$

where matrix $\boldsymbol{K} \in I\!R^{r\times n}$ is the controller gain matrix. Accepting a network delay-time, the event-time-driven closed-loop system (1), (2) takes form

$$\dot{\boldsymbol{q}}(t)=\boldsymbol{A}\boldsymbol{q}(t)+\boldsymbol{B}\boldsymbol{K}\boldsymbol{q}(i_k\Delta t),\ t\in\langle i_k\Delta t+\tau_k, j_k) \tag{4}$$

$$\dot{\boldsymbol{q}}(t)=\boldsymbol{A}\boldsymbol{q}(t), \quad t\in\langle j_k, i_{k+1}\Delta t+\tau_{k+1}) \tag{5}$$

where ($i_k$ : $k = 1, 2, \ldots$) are some integers, $\Delta t$ is the sampling period, and $\tau_k \geq 0$ is the time delay, which denotes the time interval from instant time $i_k\Delta t$ where sensors notes sample sensor data from the plat to the instant time when actuators transfer data to the plant.

It is supposed that the next condition is satisfied

$$j_k = \begin{cases} i_{k+1}\Delta t+\tau_{k+1}, & (i_{k+1}-i_k)\Delta t+\tau_{k+1} \leq h \\ i_k\Delta t + h, & (i_{k+1}-i_k)\Delta t+\tau_{k+1} > h \end{cases} \tag{6}$$

Event-time-driven mode means, that the controller and actuators data will be updated once a new packet comes, and this new data can be held during the intervening time less then $h$. If at the end of this time interval new data packet has not yet come, acting data will be set to zero a will stay zero until the new data will come. By this rule obtained switched delay system may include an unstable subsystem (see Sun et al. (2008)).

## 3. BASIC PRELIMINARIES

### 3.1 *Schur Complement*

Nonlinear convex inequalities can be converted to LMI form using Schur's complements. Let a linear matrix inequality takes form

$$\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & -\boldsymbol{R} \end{bmatrix} < 0, \qquad \boldsymbol{Q} = \boldsymbol{Q}^T,\ \boldsymbol{R} = \boldsymbol{R}^T,\ \det \boldsymbol{R} \neq 0 \tag{7}$$

Using Gauss elimination, it yields

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{S}\boldsymbol{R}^{-1} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & -\boldsymbol{R} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Q} + \boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{R} \end{bmatrix} \tag{8}$$

Since

$$\det \begin{bmatrix} \boldsymbol{I} & \boldsymbol{S}\boldsymbol{R}^{-1} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} = 1 \tag{9}$$

where $\boldsymbol{I}$ is the identity matrix of appropriate dimension, with this transform negativity of (7) is not changed, i.e. this follows as a consequence

$$\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & -\boldsymbol{R} \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} \boldsymbol{Q} + \boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{R} \end{bmatrix} < 0 \Updownarrow \boldsymbol{Q} + \boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T < 0, \quad \boldsymbol{R} > 0 \tag{10}$$

respectively. As one can see, this complement offer possibility to rewrite nonlinear inequalities in a closed matrix LMI form (see e.g. Boyd at al. (1994), Krokavec and Filasová (2008)).

### 3.2 *Null complement*

Since from the Leibniz–Newton formula

$$\int_{t-\tau}^{t} \dot{\boldsymbol{x}}(r)\mathrm{d}r = \boldsymbol{x}(t) - \boldsymbol{x}(t-\tau) \tag{11}$$

implies

$$\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau) - \int_{t-\tau}^{t} \dot{\boldsymbol{x}}(r)\mathrm{d}r = 0 \tag{12}$$

it is evident that for any nonzero matrix $\boldsymbol{W}$ of appropriate dimension is

$$\boldsymbol{z}^T(t)\boldsymbol{W}\left[\boldsymbol{x}(t)-\boldsymbol{x}(t-\tau)-\int_{t-\tau}^{t} \dot{\boldsymbol{x}}(r)\mathrm{d}r\right] = 0 \tag{13}$$

where $\boldsymbol{z}(t)$ is an arbitrary vector (see Park (1999), Krokavec and Filasová (2007)).

### 3.3 *Symmetric upper-bounds inequality*

Let $f(\boldsymbol{x}(r), v)$, $\boldsymbol{x}(r) \in I\!R^n$, $\boldsymbol{X} > 0$, $\boldsymbol{X} \in I\!R^{n \times n}$, $a > 0$, $a \in I\!R$, is a real positive definite and integrable vector function of the form

$$f(\boldsymbol{x}(r), v) = \boldsymbol{x}^T(r) e^{av} \boldsymbol{X} \boldsymbol{x}(r) \tag{14}$$

such, that there exists well defined integration as following

$$\int_{-b}^{0} \int_{t+v}^{t} f(\boldsymbol{x}(r), v) \mathrm{d}r \mathrm{d}v > 0 \tag{15}$$

where $b \geq 0$, $b \in I\!R$, $t \in \langle 0, \infty)$.

Since for (14) one can write

$$\boldsymbol{x}^T(r) e^{av} \boldsymbol{X} \boldsymbol{x}(r) - \boldsymbol{x}^T(r) e^{av} \boldsymbol{X} \boldsymbol{x}(r) = 0 \tag{16}$$

thus, by Schur complement it is true, that

$$\begin{bmatrix} \boldsymbol{x}^T(r) e^{av} \boldsymbol{X} \boldsymbol{x}(r) & \boldsymbol{x}^T(r) \\ \boldsymbol{x}(r) & e^{-av} \boldsymbol{X}^{-1} \end{bmatrix} \geq 0 \tag{17}$$

and double integration of (14) leads to

$$\begin{bmatrix} \int_{-b}^{0}\int_{t+v}^{t} \boldsymbol{x}^T(r) e^{av} \boldsymbol{X} \boldsymbol{x}(r) \mathrm{d}r \mathrm{d}v & \int_{-b}^{0}\int_{t+v}^{t} \boldsymbol{x}^T(r) \mathrm{d}r \mathrm{d}v \\ \int_{-b}^{0}\int_{t+v}^{t} \boldsymbol{x}(r) \mathrm{d}r \mathrm{d}v & \int_{-b}^{0}\int_{t+v}^{t} e^{-av} \boldsymbol{X}^{-1} \mathrm{d}r \mathrm{d}v \end{bmatrix} \geq 0 \tag{18}$$

Then with

$$\int_{t+v}^{t} e^{-av} \boldsymbol{X}^{-1} \mathrm{d}r = -v e^{-av} \boldsymbol{X}^{-1} \tag{19}$$

and with

$$\begin{aligned} &\int_{-b}^{0} -v e^{-av} \boldsymbol{X}^{-1} \mathrm{d}v = \\ &= \frac{v}{a} e^{-av} \boldsymbol{X}^{-1} \Big|_{-b}^{0} - \int_{-b}^{0} \frac{1}{a} e^{-av} \boldsymbol{X}^{-1} \mathrm{d}v = \\ &= \frac{1}{a^2} (va + 1) e^{-av} \boldsymbol{X}^{-1} \Big|_{-b}^{0} = c^{-1} \boldsymbol{X}^{-1} \end{aligned} \tag{20}$$

where

$$\boldsymbol{c}^{-1} = \frac{1}{a^2} (1 + ab e^{ab} - e^{-ab}) \tag{21}$$

inequality (18) can be rewritten as

$$\begin{bmatrix} \int_{-b}^{0}\int_{t+v}^{t} \boldsymbol{x}^T(r) e^{av} \boldsymbol{X} \boldsymbol{x}(r) \mathrm{d}r \mathrm{d}v & * \\ \int_{-b}^{0}\int_{t+v}^{t} \boldsymbol{x}(r) \mathrm{d}r \mathrm{d}v & c^{-1} \boldsymbol{X}^{-1} \end{bmatrix} \geq 0 \tag{22}$$

It is evident, that (22) implies

$$\begin{aligned} &\int_{-b}^{0}\int_{t+v}^{t} \boldsymbol{x}^T(r) e^{av} \boldsymbol{X} \boldsymbol{x}(r) \mathrm{d}r \mathrm{d}v \geq \\ &\geq \Big[ \int_{-b}^{0}\int_{t+v}^{t} \boldsymbol{x}(r) \mathrm{d}r \mathrm{d}v \Big]^T c \boldsymbol{X} \int_{-b}^{0}\int_{t+v}^{t} \boldsymbol{x}(r) \mathrm{d}r \mathrm{d}v \end{aligned} \tag{23}$$

(see e.g. Sun et al. (2008)). Hereafter, $*$ denotes the symmetric item in a symmetric matrix.

## 4. EXPONENTIAL STABILITY OF THE AUTONOMOUS SYSTEM

Defining the delay-dependent Lyapunov–Krasovskii functional as follows

$$\begin{aligned} &v(\boldsymbol{q}(t)) = \boldsymbol{q}^T(t) \boldsymbol{P} \boldsymbol{q}(t) + \\ &+ \int_{-h}^{0}\int_{t+v}^{t} \dot{\boldsymbol{q}}^T(r) e^{\alpha_1 (r-t)} \boldsymbol{R} \dot{\boldsymbol{q}}(r) \mathrm{d}r \mathrm{d}v > 0 \end{aligned} \tag{24}$$

where $\boldsymbol{P} = \boldsymbol{P}^T > 0$, and $\boldsymbol{R} = \boldsymbol{R}^T > 0$, and evaluating derivative of $v(\boldsymbol{q}(t))$ one obtains

$$\begin{aligned} &\dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t) \boldsymbol{P} \boldsymbol{q}(t) + \boldsymbol{q}^T(t) \boldsymbol{P} \dot{\boldsymbol{q}}(t) + \\ &+ h \int_{t}^{t} \dot{\boldsymbol{q}}^T(r) e^{\alpha_1 0} \boldsymbol{R} \dot{\boldsymbol{q}}(r) dr - \\ &- \int_{t-h}^{t} \dot{\boldsymbol{q}}^T(r) e^{-\alpha_1 h} \boldsymbol{R} \dot{\boldsymbol{q}}(r) dr - \\ &- \alpha_1 \int_{-h}^{0}\int_{t+v}^{t} \dot{\boldsymbol{q}}^T(r) e^{\alpha_1 (r-t)} \boldsymbol{R} \dot{\boldsymbol{q}}(r) \mathrm{d}r \mathrm{d}v < 0 \end{aligned} \tag{25}$$

$$\begin{aligned} &\dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t) \boldsymbol{P} \boldsymbol{q}(t) + \boldsymbol{q}^T(t) \boldsymbol{P} \dot{\boldsymbol{q}}(t) + \\ &+ h \dot{\boldsymbol{q}}^T(t) \boldsymbol{R} \dot{\boldsymbol{q}}(t) - \\ &- \int_{t-h}^{t} \dot{\boldsymbol{q}}^T(r) e^{-\alpha_1 h} \boldsymbol{R} \dot{\boldsymbol{q}}(r) dr - \\ &- \alpha_1 \int_{-h}^{0}\int_{t+v}^{t} \dot{\boldsymbol{q}}^T(r) e^{\alpha_1 (r-t)} \boldsymbol{R} \dot{\boldsymbol{q}}(r) \mathrm{d}r \mathrm{d}v < 0 \end{aligned} \tag{26}$$

respectively. Therefore, it follows as a consequence

$$\begin{aligned}&\dot{v}(\boldsymbol{q}(t)) - \alpha_2 v(\boldsymbol{q}(t)) = \\ &= \boldsymbol{q}^T(t)(\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{h}\boldsymbol{A}^T\boldsymbol{R}\boldsymbol{A}-\alpha_2\boldsymbol{P})\boldsymbol{q}(t)- \\ &- \int_{t-h}^{t} \dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r)dr- \\ &-(\alpha_1+\alpha_2)\dot{v}^{\circ}(\boldsymbol{q}(t)) < 0\end{aligned} \tag{27}$$

where

$$\begin{aligned}&\dot{v}^{\circ}(\boldsymbol{q}(t)) = \\ &= \Big[\int_{-h}^{0}\int_{t+v}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r\mathrm{d}v\Big]^T c\boldsymbol{R}\int_{-h}^{0}\int_{t+v}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r\mathrm{d}v\end{aligned} \tag{28}$$

$$\boldsymbol{c}^{-1} = \frac{1}{\alpha_1^2}(1+\alpha_1 h e^{\alpha_1 h} - e^{-\alpha_1 h}) \tag{29}$$

Since it can be written

$$\begin{aligned}&\int_{-h}^{0}\int_{t+v}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r\mathrm{d}v = \int_{-h}^{0}\big(\boldsymbol{q}(t)-\boldsymbol{q}(t+v)\big)\mathrm{d}v = \\ &= h\boldsymbol{q}(t) - \int_{-h}^{0}\boldsymbol{q}(t+v)\mathrm{d}v = h\boldsymbol{q}(t) - \int_{t-h}^{t}\boldsymbol{q}(r)\mathrm{d}r\end{aligned} \tag{30}$$

using (23) an upper bound $-\dot{v}^{\bullet}(\boldsymbol{q}(t))$ of $-\dot{v}^{\circ}(\boldsymbol{q}(t))$ is

$$\begin{aligned}&-\dot{v}^{\bullet}(\boldsymbol{q}(t)) = \\ &= -\big[h\boldsymbol{q}(t) - \int_{t-h}^{t}\boldsymbol{q}(r)\mathrm{d}r\big]^T \boldsymbol{cR}\,\big[h\boldsymbol{q}(t) - \int_{t-h}^{t}\boldsymbol{q}(r)\mathrm{d}r\big]\end{aligned} \tag{31}$$

and with notation

$$\boldsymbol{p}^T(t) = \Big[\,\boldsymbol{q}^T(t)\ \ \boldsymbol{q}^T(t-h)\ \ \int_{t-h}^{t}\boldsymbol{q}^T(r)\mathrm{d}r\,\Big] \tag{32}$$

(31) can be rewritten as

$$\begin{aligned}&-\dot{v}^{\bullet}(\boldsymbol{q}(t)) = \\ &= -\boldsymbol{p}^T(t)\begin{bmatrix} h \\ 0 \\ -1\end{bmatrix}\boldsymbol{cR}\begin{bmatrix} h & 0 & -1\end{bmatrix}\boldsymbol{p}(t)\end{aligned} \tag{33}$$

By the same way, using (32), constraint (13) can be adapted for solution in the next form

$$\begin{aligned}&0 = \boldsymbol{p}^T(t)\boldsymbol{W}\Big[\boldsymbol{q}(t) - \boldsymbol{q}(t-h) - \int_{t-h}^{t}\dot{\boldsymbol{q}}(r)dr\Big] + \\ &+\Big[\boldsymbol{q}^T(t) - \boldsymbol{q}^T(t-h) - \int_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)dr\Big]^T\boldsymbol{W}^T\boldsymbol{p}(t)\end{aligned} \tag{34}$$

$$\begin{aligned}&0 = \boldsymbol{p}^T(t)\boldsymbol{W}\begin{bmatrix} 1 & -1 & 0\end{bmatrix}\boldsymbol{p}(t) + \\ &+\boldsymbol{p}^T(t)\begin{bmatrix} 1 & -1 & 0\end{bmatrix}^T\boldsymbol{W}^T\boldsymbol{p}(t) - \\ &-\boldsymbol{p}^T(t)\boldsymbol{W}\int_{t-h}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r - \int_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)\mathrm{d}r\,\boldsymbol{W}^T\boldsymbol{p}(t)\end{aligned} \tag{35}$$

respectively, where

$$\boldsymbol{W}^T = \begin{bmatrix} \boldsymbol{W}_1^T & \boldsymbol{W}_2^T & \boldsymbol{W}_3^T\end{bmatrix} \tag{36}$$

Therefore, with (33) and (36), inequality (28) can be rewritten into the form

$$\begin{aligned}&\dot{v}(\boldsymbol{q}(t)) - \alpha_2 v(\boldsymbol{q}(t)) \le \boldsymbol{q}^T(t)\boldsymbol{S}^{\circ}\boldsymbol{q}(t) + \\ &+\boldsymbol{p}^T(t)\boldsymbol{T}^{\circ}\boldsymbol{p}(t) - \int_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r)dr - \\ &-\boldsymbol{p}^T(t)\boldsymbol{W}\int_{t-h}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r - \int_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)\mathrm{d}r\,\boldsymbol{W}^T\boldsymbol{p}(t) < 0\end{aligned} \tag{37}$$

where

$$\boldsymbol{S}^{\circ} = \boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{h}\boldsymbol{A}^T\boldsymbol{R}\boldsymbol{A}-\alpha_2\boldsymbol{P} \tag{38}$$

$$\begin{aligned}&\boldsymbol{T}^{\circ} = -(\alpha_1+\alpha_2)\begin{bmatrix} h \\ 0 \\ -1\end{bmatrix}\boldsymbol{cR}\begin{bmatrix} h & 0 & -1\end{bmatrix} + \\ &+\boldsymbol{W}\begin{bmatrix} 1 & -1 & 0\end{bmatrix} + \begin{bmatrix} 1 & -1 & 0\end{bmatrix}^T\boldsymbol{W}^T\end{aligned} \tag{39}$$

Since

$$\boldsymbol{q}^T(t)\boldsymbol{S}^{\circ}\boldsymbol{q}(t) = \boldsymbol{p}^T(t)\boldsymbol{T}^{\diamond}\boldsymbol{p}(t) \tag{40}$$

with

$$\boldsymbol{T}^{\diamond} = \begin{bmatrix} \boldsymbol{S}^{\circ} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix} \tag{41}$$

and

$$\begin{aligned}&\boldsymbol{T}^{\circ} = \begin{bmatrix} -h^2 d\boldsymbol{R} & \boldsymbol{0} & hd\boldsymbol{R} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ hd\boldsymbol{R} & \boldsymbol{0} & -d\boldsymbol{R}\end{bmatrix} + \\ &+\begin{bmatrix} \boldsymbol{W}_1+\boldsymbol{W}_1^T & -\boldsymbol{W}_1+\boldsymbol{W}_2^T & \boldsymbol{W}_3^T \\ \boldsymbol{W}_2-\boldsymbol{W}_1^T & -\boldsymbol{W}_2-\boldsymbol{W}_2^T & -\boldsymbol{W}_3^T \\ \boldsymbol{W}_3 & -\boldsymbol{W}_3 & \boldsymbol{0}\end{bmatrix}\end{aligned} \tag{42}$$

with

$$d = (\alpha_1+\alpha_2)c \tag{43}$$

it is evident, that

$$\begin{aligned}&\dot{v}(\boldsymbol{q}(t)) - \alpha_2 v(\boldsymbol{q}(t)) \le \boldsymbol{p}^T(t)\boldsymbol{T}^{\bullet}\boldsymbol{p}(t) - \\ &-\boldsymbol{p}^T(t)\boldsymbol{W}\int_{t-h}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r - \int_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)\mathrm{d}r\boldsymbol{W}^T\boldsymbol{p}(t) - \\ &-\int_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r)dr < 0\end{aligned} \tag{44}$$

where

$$\boldsymbol{T}^{\bullet}=\begin{bmatrix}\boldsymbol{S} & -\boldsymbol{W}_1+\boldsymbol{W}_2^T & -hd\boldsymbol{R}+\boldsymbol{W}_3^T\\ * & -\boldsymbol{W}_2-\boldsymbol{W}_2^T & -\boldsymbol{W}_3^T\\ * & * & -d\boldsymbol{R}\end{bmatrix} \quad (45)$$

$$\begin{aligned}\boldsymbol{S}=\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{h}\boldsymbol{A}^T\boldsymbol{R}\boldsymbol{A}-\\ -\alpha_2\boldsymbol{P}-h^2d\boldsymbol{R}+\boldsymbol{W}_1+\boldsymbol{W}_1^T\end{aligned} \quad (46)$$

Analogously to (17) it yields

$$\begin{bmatrix}\dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r) & \dot{\boldsymbol{q}}^T(r)\\ \dot{\boldsymbol{q}}(r) & e^{\alpha_1 h}\boldsymbol{R}^{-1}\end{bmatrix}\geq 0 \quad (47)$$

and since

$$\int\limits_{t-h}^{t} e^{\alpha_1 h}\boldsymbol{R}^{-1}dt = he^{\alpha_1 h}\boldsymbol{R}^{-1} \quad (48)$$

one can write

$$\begin{bmatrix}\int\limits_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r)\mathrm{d}r & \int\limits_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)\mathrm{d}r\\ \int\limits_{t-h}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r & he^{\alpha_1 h}\boldsymbol{R}^{-1}\end{bmatrix}\geq 0 \quad (49)$$

$$\begin{aligned}&\int\limits_{t-h}^{r}\dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r)dr\geq\\ &\geq\int\limits_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)\mathrm{d}r\big(h^{-1}e^{-\alpha_1 h}\boldsymbol{R}\big)\int\limits_{t-h}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r\end{aligned} \quad (50)$$

Thus, using upper bound (50), it results in

$$\begin{aligned}&-\int\limits_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r)dr-\\ &-\boldsymbol{p}^T(t)\boldsymbol{W}\int\limits_{t-h}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r-\int\limits_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)\mathrm{d}r\boldsymbol{W}^T\boldsymbol{p}(t)\leq\\ &\leq\dot{\boldsymbol{p}}^T(t)\begin{bmatrix}0\\0\\-1\end{bmatrix}\boldsymbol{Z}\begin{bmatrix}0\\0\\-1\end{bmatrix}^T\dot{\boldsymbol{p}}(t)+\\ &+\boldsymbol{p}^T(t)\boldsymbol{W}\begin{bmatrix}0\\0\\-1\end{bmatrix}^T\dot{\boldsymbol{p}}(t)+\dot{\boldsymbol{p}}^T(t)\begin{bmatrix}0\\0\\-1\end{bmatrix}\boldsymbol{W}^T\boldsymbol{p}(t)=\\ &=\dot{v}^{\diamond}(t)\end{aligned} \quad (51)$$

where

$$\boldsymbol{Z}=-h^{-1}e^{-\alpha_1 h}\boldsymbol{R} \quad (52)$$

Completing to square for $\boldsymbol{Z}$ gives

$$\begin{aligned}&\dot{v}^{\diamond}(t)=-\boldsymbol{p}^T(t)\boldsymbol{W}\boldsymbol{Z}^{-1}\boldsymbol{W}^T\boldsymbol{p}(t)+\\ &+\Big[\dot{\boldsymbol{p}}^T(t)\begin{bmatrix}0\\0\\1\end{bmatrix}\boldsymbol{Z}+\boldsymbol{p}^T(t)\boldsymbol{W}\Big]\boldsymbol{Z}^{-1}\Big[\boldsymbol{W}^T\boldsymbol{p}(t)+\boldsymbol{Z}\begin{bmatrix}0\\0\\1\end{bmatrix}^T\dot{\boldsymbol{p}}(t)\Big]=\\ &=-\boldsymbol{p}^T(t)\boldsymbol{W}\boldsymbol{Z}^{-1}\boldsymbol{W}^T\boldsymbol{p}(t)+\theta(t)\end{aligned} \quad (53)$$

Since for $\boldsymbol{Z}<0$ is $\theta(t)<0$, it is obvious, that

$$\begin{aligned}&\dot{v}(\boldsymbol{q}(t))-\alpha_2 v(\boldsymbol{q}(t))\leq+\theta(t)+\\ &+\boldsymbol{p}^T(t)\boldsymbol{T}^{\bullet}\boldsymbol{p}(t)+\boldsymbol{p}^T(t)\boldsymbol{W}\boldsymbol{Z}^{-1}\boldsymbol{W}^T\boldsymbol{p}(t)<0\end{aligned} \quad (54)$$

is negative, if

$$\boldsymbol{T}=\boldsymbol{T}^{\bullet}+\boldsymbol{W}\boldsymbol{Z}^{-1}\boldsymbol{W}^T<0 \quad (55)$$

Using Schur complement property with (36) and (45), inequality (55) can now be rewritten as follows

$$\begin{aligned}&\boldsymbol{T}=\\ &=\begin{bmatrix}\boldsymbol{S} & -\boldsymbol{W}_1+\boldsymbol{W}_2^T & -hd\boldsymbol{R}+\boldsymbol{W}_3^T & h\boldsymbol{W}_1\\ * & -\boldsymbol{W}_2-\boldsymbol{W}_2^T & -\boldsymbol{W}_3^T & h\boldsymbol{W}_2\\ * & * & -d\boldsymbol{R} & h\boldsymbol{W}_3\\ * & * & * & e^{-\alpha_1 h}\boldsymbol{R}\end{bmatrix}<0\end{aligned} \quad (56)$$

$$\begin{aligned}\boldsymbol{S}=\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{h}\boldsymbol{A}^T\boldsymbol{R}\boldsymbol{A}-\\ -\alpha_2\boldsymbol{P}-h^2d\boldsymbol{R}+\boldsymbol{W}_1+\boldsymbol{W}_1^T\end{aligned} \quad (57)$$

Given matrices $\boldsymbol{P}>0$, $\boldsymbol{R}>0$, and scalar $\alpha_1>0$, $h>0$, then $\dot{v}(\boldsymbol{q}(t))-\alpha_2 v(\boldsymbol{q}(t))$ is negative, if there exist scalar $\alpha_2>0$ and matrices $\boldsymbol{W}_i,\ i=1,2,3$ such that (56) holds. Therefore it also holds

$$e^{-\alpha_2 t}\dot{v}(\boldsymbol{q}(t))-e^{-\alpha_2 t}\alpha_2 v(\boldsymbol{q}(t))<0 \quad (58)$$

(compare with Sun et al. (2008)). Integrating (58) from 0 to $t$ one obtains

$$\begin{aligned}&\int\limits_{0}^{t}e^{-\alpha_2 r}\dot{v}(\boldsymbol{q}(r))\mathrm{d}r-\int\limits_{0}^{t}e^{-\alpha_2 r}\alpha_2 v(\boldsymbol{q}(r))\mathrm{d}r=\\ &=e^{-\alpha_2 r}v(\boldsymbol{q}(r))\big|_0^t=\\ &=e^{-\alpha_2 t}v(\boldsymbol{q}(t))-v(\boldsymbol{q}(0))<0\end{aligned} \quad (59)$$

$$v(\boldsymbol{q}(t))<e^{\alpha_2 t}v(\boldsymbol{q}(0)) \quad (60)$$

respectively. It is obvious, that with this conditions a trajectory of an autonomous system is stable.

## 5. EXPONENTIAL STABILITY OF THE CONTROLLED SYSTEM

Since in this case the derivative of Lyapunov–Krasovskii functional also takes form (26), then it implies

$$\begin{aligned}&\dot{v}(\boldsymbol{q}(t))+\alpha_1 v(\boldsymbol{q}(t))=\alpha_1\boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{q}(t)-\\ &-\dot{\boldsymbol{q}}^T(t)\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{q}^T(t)\boldsymbol{P}\dot{\boldsymbol{q}}(t)+\\ &+h\dot{\boldsymbol{q}}^T(t)\boldsymbol{R}\dot{\boldsymbol{q}}(t)-\\ &-\int\limits_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r)dr<0\end{aligned} \quad (61)$$

With known matrix $\boldsymbol{K}$ of the control law (4) one can write

$$\begin{aligned}&\dot{\boldsymbol{q}}^T(t)\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{q}^T(t)\boldsymbol{P}\dot{\boldsymbol{q}}(t)=\\&=\boldsymbol{q}^T(t)(\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A})\boldsymbol{q}(t)+\\&+\boldsymbol{q}^T(t-\tau)\boldsymbol{K}^T\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{q}(t)+\\&+\boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{B}\boldsymbol{K}\boldsymbol{q}^T(t-\tau)\end{aligned} \tag{62}$$

It is evident, that for any $\theta(t)\geq 0$, inequality (61) can be rewritten as follows

$$\begin{aligned}&\dot{v}(\boldsymbol{q}(t))+\alpha_1 v(\boldsymbol{q}(t))\leq\alpha_1\boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{q}(t)-\\&-\dot{\boldsymbol{q}}^T(t)\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{q}^T(t)\boldsymbol{P}\dot{\boldsymbol{q}}(t)+\\&+h\dot{\boldsymbol{q}}^T(t)\boldsymbol{R}\dot{\boldsymbol{q}}(t)-\\&-\int_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)e^{-\alpha_1 h}\boldsymbol{R}\dot{\boldsymbol{q}}(r)dr+0+\theta(t)<0\end{aligned} \tag{63}$$

Defining

$$\boldsymbol{v}^T(t)=\left[\,\boldsymbol{q}(t)^T\;\;\boldsymbol{q}^T(t-h)\,\right] \tag{64}$$

$$\boldsymbol{s}^T(t,r)=\left[\,\boldsymbol{q}^T(t)\;\;\boldsymbol{q}^T(t-h)\;\;\dot{\boldsymbol{q}}^T(r)\,\right] \tag{65}$$

then, using (13) it holds

$$\begin{aligned}0=&\,\boldsymbol{v}^T(t)\begin{bmatrix}\boldsymbol{V}_1\\\boldsymbol{V}_2\end{bmatrix}\Big[\boldsymbol{q}(t)-\boldsymbol{q}(t-h)-\int_{t-h}^{t}\dot{\boldsymbol{q}}(r)dr\Big]+\\&+\Big[\boldsymbol{q}(t)-\boldsymbol{q}(t-h)-\int_{t-h}^{t}\dot{\boldsymbol{q}}(r)dr\Big]^T\begin{bmatrix}\boldsymbol{V}_1\\\boldsymbol{V}_2\end{bmatrix}^T\boldsymbol{v}(t)\end{aligned} \tag{66}$$

$$\begin{aligned}&0=\boldsymbol{v}^T(t)\boldsymbol{V}\left[\,1\;-1\;\;0\,\right]\boldsymbol{v}(t)+\\&+\boldsymbol{v}^T(t)\left[\,1\;-1\;\;0\,\right]^T\boldsymbol{V}^T\boldsymbol{v}(t)-\\&-\boldsymbol{v}^T(t)\boldsymbol{V}\int_{t-h}^{t}\dot{\boldsymbol{q}}(r)\mathrm{d}r-\int_{t-h}^{t}\dot{\boldsymbol{q}}^T(r)\mathrm{d}r\,\boldsymbol{V}^T\boldsymbol{v}(t)\end{aligned} \tag{67}$$

respectively, where

$$\boldsymbol{V}^T=\left[\,\boldsymbol{V}_1^T\;\;\boldsymbol{V}_2^T\,\right] \tag{68}$$

$$\begin{aligned}&\boldsymbol{v}^T(t)\boldsymbol{V}\left[\,1\;-1\;\;0\,\right]\boldsymbol{v}(t)+\\&+\boldsymbol{v}^T(t)\left[\,1\;-1\;\;0\,\right]^T\boldsymbol{V}^T\boldsymbol{v}(t)=\\&=\boldsymbol{v}^T(t)\boldsymbol{U}^{\circ}\boldsymbol{v}(t)\end{aligned} \tag{69}$$

$$\begin{aligned}\boldsymbol{U}^{\circ}&=\boldsymbol{V}\left[\,1\;-1\;\;0\,\right]+\left[\,1\;-1\;\;0\,\right]^T\boldsymbol{V}^T=\\&=\begin{bmatrix}\boldsymbol{V}_1+\boldsymbol{V}_1^T&-\boldsymbol{V}_1+\boldsymbol{V}_2^T\\ *&-\boldsymbol{V}_2-\boldsymbol{V}_2^T\end{bmatrix}\end{aligned} \tag{70}$$

On the other hand, for $h>0$ and any semi-positive definite matrix $\boldsymbol{Q}\geq 0$, it is true

$$\begin{aligned}\vartheta(t)&=h\boldsymbol{v}^T(t)\boldsymbol{Q}\boldsymbol{v}(t)-h\boldsymbol{v}^T(t)\boldsymbol{Q}\boldsymbol{v}(t)=\\&=h\boldsymbol{v}^T(t)\boldsymbol{Q}\boldsymbol{v}(t)-\int_{t-h}^{t}\boldsymbol{v}^T(t)\boldsymbol{Q}\boldsymbol{v}(t)\mathrm{d}r=0\end{aligned} \tag{71}$$

where

$$\boldsymbol{Q}=\begin{bmatrix}\boldsymbol{Q}_{11}&\boldsymbol{Q}_{12}\\ *&\boldsymbol{Q}_{22}\end{bmatrix} \tag{72}$$

Since it is possible to write

$$\begin{aligned}&h\dot{\boldsymbol{q}}^T(t)\boldsymbol{R}\dot{\boldsymbol{q}}(t)=\\&=\boldsymbol{v}^T(t)\begin{bmatrix}\boldsymbol{A}^T\\\boldsymbol{K}^T\boldsymbol{B}^T\end{bmatrix}h\boldsymbol{R}\big[\boldsymbol{A}\;\;\boldsymbol{K}\boldsymbol{B}\big]\boldsymbol{v}(t)=\\&=\boldsymbol{v}^T(t)\boldsymbol{U}^{\diamond}\boldsymbol{v}(t)\end{aligned} \tag{73}$$

$$\begin{aligned}&\boldsymbol{U}^{\diamond}=\\&=\begin{bmatrix}h\boldsymbol{A}^T\boldsymbol{R}\\h\boldsymbol{K}^T\boldsymbol{B}^T\boldsymbol{R}\end{bmatrix}(h\boldsymbol{R})^{-1}\big[h\boldsymbol{R}\boldsymbol{A}\;\;h\boldsymbol{R}\boldsymbol{K}\boldsymbol{B}\big]\end{aligned} \tag{74}$$

and since in this regime the constraint can be adapted for solution in the structure (67), then one can combine elements in integrals as follows

$$\begin{aligned}&\boldsymbol{v}^T(t)\boldsymbol{Q}\boldsymbol{v}(t)+\dot{\boldsymbol{q}}^T(r)e_1^{-\alpha}h\boldsymbol{R}\dot{\boldsymbol{q}}(r)-\\&-\dot{\boldsymbol{q}}^T(r)\boldsymbol{V}^T\boldsymbol{v}(t)-\boldsymbol{v}^T(t)\boldsymbol{V}\dot{\boldsymbol{q}}(r)=\\&=\boldsymbol{s}^T(t,r)\boldsymbol{Q}^{\bullet}\boldsymbol{s}(t,r)\end{aligned} \tag{75}$$

$$\boldsymbol{Q}^{\bullet}=\begin{bmatrix}\boldsymbol{Q}_{11}&\boldsymbol{Q}_{12}&-\boldsymbol{V}_1\\ *&\boldsymbol{Q}_{22}&-\boldsymbol{V}_2\\ *&*&e^{-\alpha_1 h}\boldsymbol{R}\end{bmatrix} \tag{76}$$

Thus, with notation,

$$\boldsymbol{q}^T(t)(\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A})\boldsymbol{q}(t)=\boldsymbol{v}^T(t)\boldsymbol{U}^{\triangleright}\boldsymbol{v}(t) \tag{77}$$

$$\boldsymbol{U}^{\triangleright}=\begin{bmatrix}\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}&\boldsymbol{0}\\ *&\boldsymbol{0}\end{bmatrix} \tag{78}$$

inequality (63) can be written in the form

$$\begin{aligned}&\dot{v}(\boldsymbol{q}(t))+\alpha_1 v(\boldsymbol{q}(t))\leq\boldsymbol{v}^T(t)\boldsymbol{U}^{\bullet}\boldsymbol{v}(t)-\\&-\int_{t-h}^{t}\boldsymbol{s}^T(t,r)\boldsymbol{Q}^{\bullet}\boldsymbol{s}(t,r)dr<0\end{aligned} \tag{79}$$

where

$$\boldsymbol{U}^{\bullet}=\boldsymbol{U}^{\circ}+\boldsymbol{U}^{\diamond}+\boldsymbol{U}^{\triangleright}=\begin{bmatrix}\boldsymbol{U}_{11}&\boldsymbol{U}_{12}&\boldsymbol{U}_{13}\\ *&\boldsymbol{U}_{22}&\boldsymbol{U}_{23}\\ *&*&\boldsymbol{U}_{33}\end{bmatrix} \tag{80}$$

$$\begin{aligned}&\boldsymbol{U}_{11}=\\&=\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{V}_1+\boldsymbol{V}_1^T h\boldsymbol{Q}_{11}+\alpha_1\boldsymbol{P}\end{aligned} \tag{81}$$

$$\boldsymbol{U}_{12}=\boldsymbol{P}\boldsymbol{B}\boldsymbol{K}-\boldsymbol{V}_1+\boldsymbol{V}_2^T+h\boldsymbol{Q}_{12} \tag{82}$$

$$\boldsymbol{U}_{22}=-\boldsymbol{V}_2-\boldsymbol{V}_2^T+h\boldsymbol{Q}_{22} \tag{83}$$

It is evident, that for given constant $\alpha_1>0$, $h>0$ and matrix $\boldsymbol{K}$ system is stable, if there exist matrice $\boldsymbol{P}>0$, $\boldsymbol{R}>0$ and $\boldsymbol{Q}>0$, as well as $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ such that

$$\boldsymbol{U}^{\bullet}<0,\qquad\boldsymbol{Q}^{\bullet}\geq 0 \tag{84}$$

Therefore it holds

$$e^{\alpha_1 t}\dot{v}(\boldsymbol{q}(t)) - e^{\alpha_1 t}\alpha_1 v(\boldsymbol{q}(t)) < 0 \tag{85}$$

Integrating (85) from 0 to $t$ one obtains

$$\begin{aligned} \int_0^t e^{\alpha_1 r}\dot{v}(\boldsymbol{q}(r))\mathrm{d}r + \int_0^t e^{\alpha_1 r}\alpha_1 v(\boldsymbol{q}(r))\mathrm{d}r = \\ = e^{\alpha_1 r} v(\boldsymbol{q}(r))|_0^t = \\ = e^{\alpha_1 t} v(\boldsymbol{q}(t)) - v(\boldsymbol{q}(0)) < 0 \end{aligned} \tag{86}$$

$$v(\boldsymbol{q}(t)) < e^{-\alpha_1 t} v(\boldsymbol{q}(0)) \tag{87}$$

respectively. It is obvious, that with this conditions a trajectory of the controlled system is stable.

## 6. OPTIMIZATION

Solving (84) with (76), (80) one can obtain $h$. Then according (56) it is possible to compute $\alpha_2$ and to approximate intervening time $h^\circ$ as follows

$$h^\circ = h + \frac{\alpha_1 - \alpha^\star}{\alpha_2 + \alpha^\star}(h - h_0) \tag{88}$$

where

$$0 \le \tau_k \le h_0, \quad 0 < \alpha^\star < \alpha_1 \tag{89}$$

Thus, an optimal solution can be obtained as a minimization of (56) with respect to $\alpha_2$. It is obvious, that the maximum of $h$ does not necessarily means the maximum of $h^\circ$.

Solving all matrix inequalities, i.e. (76), (80), as well as (56), one can obtain the average decay degree $\alpha_2 = 0.5\alpha^\star$, for which switched system is exponentially stable.

## 7. CONCLUDING REMARKS

This paper presents a modified method of determining delay-dependent exponential stability criteria for event–time–driven modes in networked control system. Based on linear matrix inequalities some free weighting matrix design parameters are introduced to obtain size of available rate under which system can stay exponential stable. It seems that this criteria is less conservative then existing ones.

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