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ONE-STEP ACTIVE CONTROLLER WITH DUAL PROPERTIES

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Abstract: In stochastic adaptive control, the controller that achieves required control performance and keeps gathering information about the system at the same time, is referred to as a controller with dual properties. As the optimal dual controller is computationally intractable, approximations of the optimal problem are searched. In this paper we propose a control strategy for ARX systems with dual properties. This active control strategy is based on the well known cautious strategy, but takes the quality of identification in one step ahead into consideration. This strategy shows how to improve control performance mainly in cases when the initial uncertainty in system parameters is large.

Keywords: Stochastic optimal control, cautious strategy, dual control.

1. INTRODUCTION

The goal of all control strategies is to achieve a specified performance of the controlled system. Usually, this performance is expressed in a form of a criterion, which is to be minimized. When evaluating the control strategy, it is necessary to know the structure of the controlled system. The better the knowledge is, the more effective is the control strategy. However, if the knowledge is poor, identification must be done prior to applying control algorithms. In some cases, the knowledge about the system is also improved during the control process. Here, it must be ensured that the system is sufficiently excited, so that the dentification algorithm has enough relevant data to improve the knowledge.

The control strategy that optimally solves the problem of finding a tradeoff between control performance and system identification is referred to as dual control strategy. The optimality is meant with respect to a given criterion. Unfortunately, the complete dual problem is computationally intractable even for simple systems. For this reason, various approximative strategies were designed. Filatov and Unbehauen (2004) describes the state of the art in the field and defines the properties that every dual controller must have. An overview of dual control methods can be found in Wittenmark (1995), Wittenmark (2002) and in the analysis by Lindoff et al. (1999). In this paper we propose a control strategy that is based on a one-step approximative solution of the original problem. It is based on the cautious controller described in Peterka (1986) and Wittenmark (1995), but takes into account the fact, that identification takes place in the first step of control. For the rest of the control process, cautious control is assumed. With this assumption, the controller keeps the dual property, as it optimizes the performance and identification, while remaining computationally feasible. The control strategy is derived for the ARX models that are often used to model stochastic systems (Åström and Wittenmark (1997)).

In the next section we describe in detail the N-step cautious control strategy for a general linear stochastic system. In section 3, we describe the idea of the one-step active strategy. In section 4, the general ideas are applied to the ARX model of stochastic system. Section 5 contains some simulation experiments.

2. CAUTIOUS CONTROLLER

In this section we will first describe the cautious controller for a general linear stochastic system and then use it to derive the one-step active controller in the next section. We will consider the following stochastic system:

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k) + E v(k) \\ y(k) &= C_k x(k) + D_k u(k) + e(k), \end{aligned} \tag{1}$$

where the noise vectors $v(k)$ and $e(k)$ are mutually independent white sequences with zero mean and covariances P_v and P_e, respectively. We assume perfect state information, i.e. the state $x(k)$ is perfectly known to us at time k. We also assume the system matrices to be constant linear functions of some parameter vector θ_k, i.e.

$$\begin{aligned} A_k = A(\theta_k), B_k = B(\theta_k), \\ C_k = C(\theta_k), D_k = D(\theta_k), \end{aligned} \tag{2}$$

where θ_k is a random vector. This will allow us to treat the uncertainty in system parameters in a compact way, since the system at time k now depends only on the value of the random vector θ_k.

Our goal is to find a control sequence, which will be denoted $u^*(1), \ldots, u^*(N)$, that minimizes the criterion

$$\begin{aligned} J_1^N = \mathcal{E}\left\{ \sum_{k=1}^{N} y^T(k) Q y(k) + \right. \\ \left. + u^T(k) R u(k) | x(1), u(1), \ldots, u(N) \right\}, \end{aligned} \tag{3}$$

where the symbol $\mathcal{E}\{\cdot\}$ denotes the expected value with respect to all random variables, which in this case are the unknown future outputs $y(1), \ldots, y(N)$. Q and R are constant positive definite weighting matrices that are used for tuning the controller. Generally, also time-varying matrices Q_k, R_k can be used and modification of all presented algorithms to this case is straightforward.

We will first assume the parameter vectors θ_k, $k = 1, \ldots, N$ to be independent, identically distributed (i.i.d.) random vectors with mean $\widehat{\theta}$ and covariance P_θ. This means that at every time instant k, the system matrices are i.i.d. random matrices. This assumption, however unrealistic, allows us to derive an optimal linear feedback control law, that is called the cautious control law.

The optimal value of criterion (3) depends on the initial state $x(1)$. More generally, let us denote the optimal value of criterion (3) as

$$J_k^*(x(k)) = \min_{u(k),\ldots,u(N)} J_k^N, \tag{4}$$

where the initial time is k and we omit the final time N for convenience of notation, as it remains the same in all cases. Since the criterion is additive, we can write the Bellman equation for the optimal control at time k,

$$\begin{aligned} J_k^*(x(k)) = \min_{u(k)} \mathop{\mathcal{E}}_{x(k+1),y(k)} \left\{ y^T(k) Q y(k) + \right. \\ \left. + u^T(k) R u(k) + J_{k+1}^*(x(k+1)) | u(k), x(k) \right\}, \end{aligned} \tag{5}$$

and then use the dynamic programming algorithm to derive the solution (Bertsekas (2005)).

Recall that we assume perfect state information, so the state $x(k)$ is known at time k. Also the optimal criterion value at time $k+1$ depends only on the state $x(k+1)$ and so the only uncertainties are in the output $y(k)$ and state $x(k+1)$. After substitution from (1) and recalling notation (2), equation (5) becomes

$$\begin{aligned} & J_k^*(x(k)) = \min_{u(k)} \mathop{\mathcal{E}}_{\theta_k, e(k), v(k)} \left\{ x^T(k) {C_k}^T Q C_k x(k) \right. \\ & + 2x^T(k) {C_k}^T Q D_k u(k) + \\ & + u^T(k)[R + D_k^T Q D_k] u(k) + \\ & + e^T(k)(\cdot) + e^T(k) Q e(k) + \\ & \left. + J_{k+1}^*(A_k x(k) + B_k u(k) + E v(k)) | u(k), x(k) \right\}, \end{aligned} \tag{6}$$

It will be shown by induction that the optimal criterion value at time k can be written as a quadratic form of the state $x(k)$ plus a constant term, i.e.

$$J_k^*(x(k)) = x^T(k) G_k x(k) + g_k \tag{7}$$

for some matrix G_k and a scalar g_k. It is obviously true for time $N+1$, as $J_{N+1}^*(x(N+1)) = 0$, i.e. $G_{N+1} = 0$ (zero matrix) and $g_{N+1} = 0$. If we assume that (7) is true for time $k+1$, we can substitute this expression into (6) and get the following equation for optimal criterion value at time k

$$J_k^*(x(k)) = \min_{u(k)} \mathop{\mathcal{E}}_{\theta_k,e(k),v(k)} \{x^T(k){C_k}^T QC_k x(k)$$
$$+2x^T(k){C_k}^T QD_k u(k) + \quad (8)$$
$$+u^T(k)[R + D_k^T QD_k]u(k) + e^T(k)(\cdot) +$$
$$+e^T(k)Qe(k) + [A_k x(k) + B_k u(k) + Ev(k)]^T \times$$
$$\times G_{k+1}[A_k x(k) + B_k u(k) + Ev(k)]|u(k), x(k)\},$$

and using linearity of the $\mathcal{E}\{\cdot\}$ and facts that

$$\mathcal{E}\{e(k)\} = \mathcal{E}\, v(k) = 0,$$
$$\mathcal{E}\{e^T(k)Qe(k)\} = \mathrm{tr}\,\{P_e Q\}$$

and

$$\mathcal{E}\{v^T(k)E^T G_{k+1} Ev(k)\} = \mathrm{tr}\,\{P_v E^T G_{k+1} E\},$$

where tr $\{\cdot\}$ denotes the trace operator, we get to

$$J_k^*(x(k)) = \min_{u(k)} \{x^T(k)\times \quad (9)$$
$$\times \mathop{\mathcal{E}}_{\theta_k} \{{C_k}^T QC_k + {A_k}^T G_{k+1} A_k\} x(k) +$$
$$+2x^T(k) \mathop{\mathcal{E}}_{\theta_k} \{{C_k}^T QD_k + {A_k}^T G_{k+1} B_k\} u(k) +$$
$$+u^T(k) \mathop{\mathcal{E}}_{\theta_k} \{R + D_k^T QD_k + {B_k}^T G_{k+1} B_k\} u(k) +$$
$$+ \mathrm{tr}\,\{P_e Q\} + \mathrm{tr}\,\{P_v E^T G_{k+1} E\} + g_{k+1}\}$$

By minimizing this expression with respect to $u(k)$, we get to the following expression for optimal control at time k

$$u^*(k) = - \mathop{\mathcal{E}}_{\theta_k} \{R + D_k^T QD_k + {B_k}^T G_{k+1} B_k\}^{-1} \times$$
$$\times \mathop{\mathcal{E}}_{\theta_k} \{{D_k}^T QC_k + {B_k}^T G_{k+1} A_k\} x(k), \quad (10)$$

which is a linear function of the system state $x(k)$ expressed in a closed form. Substituting (10) back into (9), we get the recursive formulas for matrix G_k and scalar g_k, which also completes the induction, as we have expressed the optimal value $J_k^*(x(k))$ in a required form (7).

$$G_k = \mathop{\mathcal{E}}_{\theta_k} \{{C_k}^T QC_k + {A_k}^T G_{k+1} A_k\} - \quad (11)$$
$$- \mathop{\mathcal{E}}_{\theta_k} \{{C_k}^T QD_k + {A_k}^T G_{k+1} B_k\} \times$$
$$\times \mathop{\mathcal{E}}_{\theta_k} \{R + D_k^T QD_k + {B_k}^T G_{k+1} B_k\}^{-1} \times$$
$$\times \mathop{\mathcal{E}}_{\theta_k} \{{D_k}^T QC_k + {B_k}^T G_{k+1} A_k\}$$
$$g_k = \mathrm{tr}\,\{P_e Q + P_v E^T G_{k+1} E\} + g_{k+1}$$

Note that random vectors θ_k are i.i.d. for all $k = 1, \ldots, N$ and so the means of expressions in (11) and (10) are the same for all $k = 1, \ldots, N$.

3. ONE-STEP ACTIVE CONTROLLER

The drawback of the cautious control strategy is the fact that the same distribution of parameter vector θ_k is assumed during the whole control process, i.e. for $k = 1, \ldots, N$. In fact, we can expect some improvement of the knowledge about the system, expressed by the change of the covariance matrix P_θ. This approach leads us to a strategy that actively improves the knowledge about the system. For this reason, we will now use the notation $\widehat{\theta}_k$ and P_{θ_k} for means and covariances of vectors θ_k.

In the case of cautious control, the optimal criterion value at time k is a function of the state $x(k)$ given by (7). The matrix of the quadratic form is given by (11). In this equation we notice that means of products of random variables are to be evaluated. This indicates that the resulting matrix G_k and therefore also the optimal criterion value $J_k^*(x(k))$ depend on the second moments of θ_k, i.e. on the covariance matrix P_{θ_k}. In the case of cautious controller we could omit this dependence, because we assumed P_{θ_k} to be a constant matrix for all $k = 1, \ldots, N$. However, if we take into account the influence of the input $u(k)$ on $P_{\theta_{k+1}}$ at time $k+1$, i.e. $P_{\theta_{k+1}} = P_{\theta_{k+1}}(u(k))$, we have to consider the matrix G_{k+1} to be a function of $u(k)$, i.e. $G_{k+1} = G_{k+1}(u(k))$ and consequently also the criterion $J_{k+1}^*(x(k+1)) = J_{k+1}^*(x(k+1), u(k))$. The Bellman equation (5) still holds, but it will now take the form

$$J_k^*(x(k)) = \min_{u(k)} \mathop{\mathcal{E}}_{x(k+1),y(k)} \{y^T(k)Qy(k)+ \quad (12)$$
$$+ u^T(k)Ru(k) +$$
$$+ J_{k+1}^*(x(k+1), u(k))|u(k), x(k)\},$$

In the one-step active control strategy, we assume that the covariance matrix P_θ is changed only after the first step of control and that cautious strategy is applied on the rest of the horizon. Under this assumption, it is possible to evaluate the criterion value $J_{k+1}^*(x(k+1), u(k))$, provided we know, how the input $u(k)$ changes the covariance P_{θ_k} in the next step, i.e. the transition from P_{θ_k} to $P_{\theta_{k+1}}$ is expressed as a function of $u(k)$. This influence is known for the ARX model of a stochastic system, where the parameters can be estimated using the appropriate form of the Kalman filter (Anderson and Moore (2005)). The minimization, however, does not lead to a closed form functions of the state $x(k)$, as in the case of cautious controller. However, it can be performed numerically, and because the optimal input for cautious control strategy is known, the optimization can be started at this point. This guarantees that the optimal solution of (12) is not worse than the cautious control.

4. ACTIVE CONTROL STRATEGY FOR ARX MODELS

In this section we apply the general principles described in the previous section to the AutoRegressive model with eXternal input (ARX) that is described by equation

$$y(k) = z^T(k)\theta_k + e(k), \tag{13}$$

where $y(k), u(k)$ are the output and the input of the system, regressor

$$z_{(k)} = [u_{(k)}, y_{(k-1)}, u_{(k-1)}, \ldots, y_{(k-n)}, u_{(k-n)}]^T$$

and

$$\theta_k = \begin{bmatrix} b_0, \; a_1, \; b_1, \; a_2, \; \ldots, \; a_n, \; b_n \end{bmatrix}^T$$

is the vector of parameters. The noise $e(k)$ is a gaussian white sequence, $e(k) \sim \mathcal{N}(0, \sigma_e^2)$. For such systems a nonminimal state representation exists, where the state vector consists of delayed inputs and outputs,

$$x(k) = [y(k-1), u(k-1), \ldots, y(k-n), u(k-n)],$$

and so the state vector is directly measurable.

The matrices of the state space description are

$$A(\theta_k) = \begin{bmatrix} a_{1,k} & b_{1,k} & \ldots & b_{n-1,k} & a_{n,k} & b_{n,k} \\ 0 & 0 & \ldots & 0 & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & \ldots & 1 & 0 & 0 \end{bmatrix},$$

$$B(\theta_k) = \begin{bmatrix} b_{0,k} \; 1 \; 0 \; \ldots \; 0 \end{bmatrix}^T,$$
$$E = \begin{bmatrix} 1 \; 0 \; \ldots \; 0 \end{bmatrix}^T,$$
$$C(\theta_k) = \begin{bmatrix} a_{1,k} \; b_{1,k} \; \ldots \; a_{n,k} \; b_{n,k} \end{bmatrix},$$
$$D(\theta_k) = b_{0,k}.$$

and finally

$$e(k) = v(k).$$

We can see that $\theta_k = \begin{bmatrix} b_0 \; C \end{bmatrix}^T$. Because parameter b_0 plays a special role in the algorithm, let us also denote the expected value $\widehat{\theta}_k$ and the submatrices of P_{θ_k} as

$$\widehat{\theta}_k = \begin{bmatrix} \widehat{b}_{0,k} \\ \widehat{C}_k \end{bmatrix}, \qquad P_{\theta_k} = \begin{bmatrix} \sigma^2_{b_0,k} & P_{b_0 C,k} \\ P_{C b_0,k} & P_{C,k} \end{bmatrix}. \tag{14}$$

We will also denote the means

$$\widehat{A}_k = A(\widehat{\theta}_k), \widehat{B}_k = B(\widehat{\theta}_k), \tag{15}$$
$$\widehat{C}_k = C(\widehat{\theta}_k), \widehat{D}_k = D(\widehat{\theta}_k),$$

Finally, we choose $Q = 1$ without loss of generality, because in the case of SISO system it is the ratio R/Q that determines uniquely the criterion. The results of the previous section applied to this ARX system have the following form:

$$u^*(k) = -\frac{l^T(k)}{\alpha(k)} x(k), \tag{16}$$

$$G_k = \widehat{C}_k^T \widehat{C}_k + P_{C,k} + \widehat{A}_k^T G_{k+1} \widehat{A}_k + G_{11,k+1} P_{C,k} - \frac{l(k) l^T(k)}{\alpha(k)}, \tag{17}$$

$$g_k = g_{k+1} + \sigma_e^2 \left(1 + G_{11,k+1}\right), \tag{18}$$

where the vector $l(k)$ equals

$$l(k) = \widehat{C}_k^T \widehat{b}_{0,k} + P_{C b_0,k} + \widehat{A}_k^T G_{k+1} \widehat{B}_k + P_{C b_0,k} G_{11,k+1}$$

and the scalar $\alpha(k)$ equals

$$\alpha(k) = R + \widehat{b}^2_{0,k} + \sigma^2_{b_0,k} + G_{11,k+1}\left(\widehat{b}^2_{0,k} + \sigma^2_{b_0,k}\right) + 2\widehat{b}_{0,k} G_{12,k+1} + G_{22,k+1}.$$

The notation $G_{ij,k}$ denotes the element in i-th row and j-th column of the matrix G_k at time k.

Recall that cautious control leads to a criterion value expressed in (7) and therefore we now have equations to evaluate the criterion value J^*_{k+1} for a given $P_{\theta_{k+1}}$:

$$J^*_{k+1}(x(k+1), u(k)) = x^T(k+1) G_{k+1}(P_{\theta_{k+1}}) x(k+1) + g_{k+1} \tag{19}$$

To find the value of $P_{\theta_{k+1}}$ for a given u_k and P_{θ_k}, we will use the Kalman filter for recursive estimation of parameters of ARX model:

$$\widehat{\theta}_{k+1} = \widehat{\theta}_k + \frac{P_{\theta_k} z(k)}{\sigma_e^2 + z^T(k) P_{\theta_k} z(k)} [y(k) - z^T(k)\widehat{\theta}_k], \tag{20}$$

which implies

$$\underset{y(k)}{\mathcal{E}} \{\widehat{\theta}_{k+1}\} = \widehat{\theta}_k \tag{21}$$

and for covariance matrix

$$P_{\theta_{k+1}} = P_{\theta_k} - \frac{P_{\theta_k} z(k) z^T(k) P_{\theta_k}}{\sigma_e^2 + z^T(k) P_{\theta_k} z(k)}. \tag{22}$$

Putting together equations (12), (19), (17), (18) and (22) we can now evaluate the input $u(1)$,

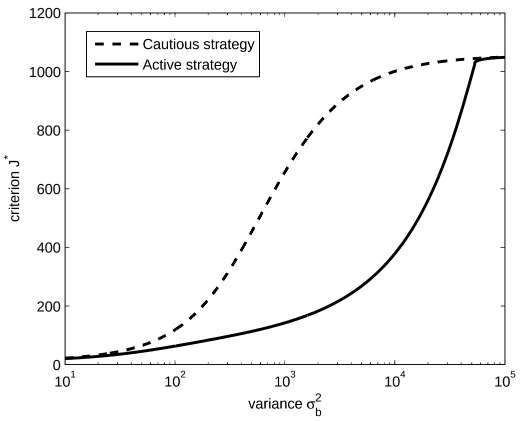

Fig. 1. The dependence of the optimal criterion value J^* on the parameter variance σ_b^2 for cautious and active control strategy.

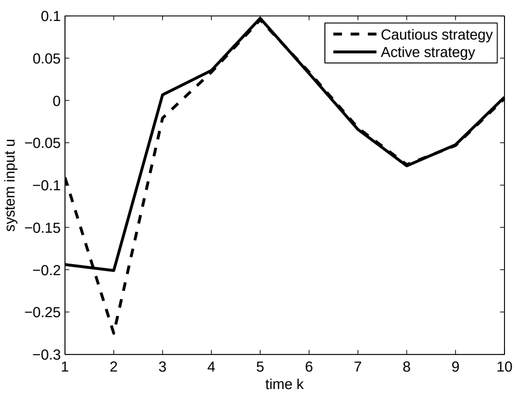

Fig. 2. Input to a controlled process with $b = 25$, while $\widehat{b} = 10$ and $\sigma_b^2 = 10^3$ is assumed.

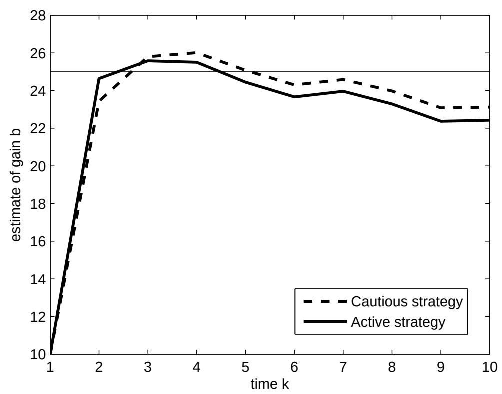

Fig. 3. The estimate of the system gain for $b = 25$, while $\widehat{b} = 10$ and $\sigma_b^2 = 10^3$ is assumed.

i.e.the first step of the active strategy by minimizing (12). Note that at time $k = 1$, everything needed for the minimization is known, as the value of $J_{k+1}^* = J_2^*$ is computed according to the assumption of cautious control. The inputs $u(2), \ldots, u(N)$ are then computed according to (16), because it is assumed that for these steps the cautious control strategy is used.

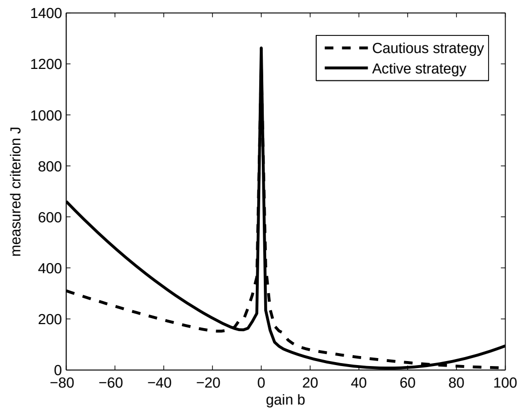

Fig. 4. Measured criterion values according to the real value of gain b, while $\widehat{b} = 10$ and $\sigma_b^2 = 10^3$ is assumed.

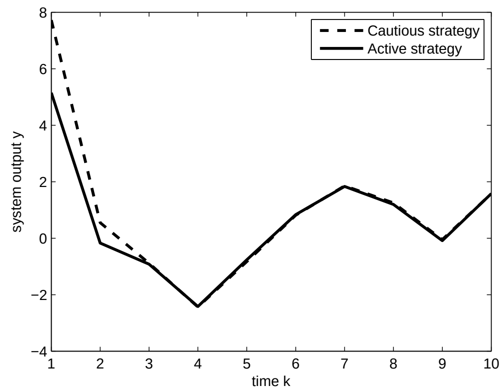

Fig. 5. Output of a controlled process with $b = 25$, while $\widehat{b} = 10$ and $\sigma_b^2 = 10^3$ is assumed.

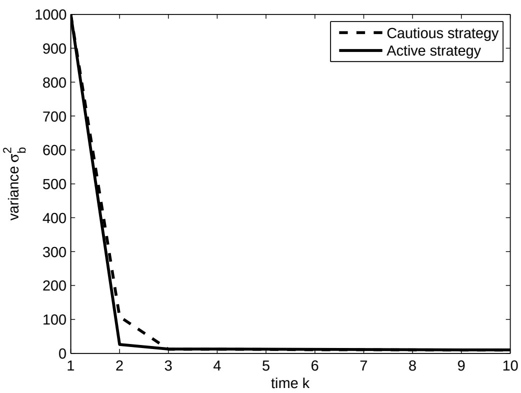

Fig. 6. The variance σ_b^2 of the estimate of the gain b with $b = 25$, while $\widehat{b} = 10$ and $\sigma_b^2 = 10^3$ is assumed.

5. SIMULATION EXPERIMENTS

In this section we present simulation experiments performed for the discrete integrator with unknown gain on the input:

$$y(k) = y(k-1) + bu(k) + e(k) \qquad (23)$$

The following values are assumed:

$$\widehat{b} = 10, \quad \sigma_e^2 = 1, \quad \sigma_b^2 \in [1, 10^5]. \tag{24}$$

The further settings are used: the number of steps for computing the criterion (the control horizon) $N = 10$, the weighting factor $R = 1$ and the initial condition $y(0) = 10$.

Figure 1 shows the optimal criterion values computed for the system (23) on the horizon of $N = 10$ steps. It compares the values of criterion for cautious and active control strategy. It can be seen that the highest difference is around the variance $\sigma_b^2 = 4 \cdot 10^3$ and that for high uncertainty, the benefit of using active strategy disappears.

For the next experiment, both control strategies (cautious and active) are applied on the system (23) in the following way. In each step of control k, the optimal input sequence over the whole horizon $N = 10$ is computed. Then only the first input $u^*(k)$ is applied and after the real output $y(k)$ is measured, the knowledge about parameter b is updated in terms of expressions (20) and (22). In the next step $k + 1$ the whole new control sequence is computed based on this improved knowledge. This makes it possible to see that the active strategy takes the identification process into account. The control process is simulated for $K = 10$ steps of control.

Figure 4 shows the dependence of the real (measured) criterion value on the real gain b. The values of parameter b are chosen in the interval $[-80, 100]$. Other settings are chosen as in (24), with $\sigma_b^2 = 10^3$. To reduce the influence of the noise e on the result, the control process is simulated 10 times for each parameter value from interval $[-80, 100]$ and the average of the criterion is taken.

Figure 2 and Figure 5 show an example of the control process with $b = 25$. The graph shows that the active strategy starts with a greater (absolute) value of control signal than the cautious one. This leads to faster parameter identification as well as faster decrease of the uncertainty expressed by σ_b^2, as can be seen in Figure 3 and Figure 6.

6. CONCLUSIONS

In this paper an adaptive control strategy is presented that shows the dual properties, while keeping computational feasibility. This active strategy is based on the cautious control strategy and assumes that uncertainty in system parameters is reduced after the first step of the control process. The theoretical values of the criterion show, that the active strategy brings the biggest benefit when the initial parameter uncertainty is high. The control process of the active strategy is simulated and compared with the cautious strategy. Experiments show that using the active strategy leads to faster parameter identification than in the case of cautious strategy. However, after several steps of control, both strategies give very similar results. The experiments were performed on a discrete integrator with unknown gain, so only one parameter was estimated and quick convergence was expectable.

As the next step, we plan to extend this one-step algorithm to a general N-step strategy, which would take into account the change of parameter knowledge in the course of N future steps of control. Thus we would come to a combination of predictive and LQ control for uncertain systems.

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