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ROBUST DESIGN OF INTEGRATING CONTROLLERS FOR IPDT PLANT

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Abstract: This paper deals with robust tuning of the Integral Plus Dead Time plant (IPDT), whereby it compares results achieved by the analytical tuning of the PI controller guaranteeing triple real dominant pole with those achieved with P controller extended by disturbance observer (DOB) under robust tuning based on experimentally achieved regions of parameters corresponding to the one-pulse control.

Keywords: Pole assignment control, Disturbance observer, Proportional control, Dead time.

1 INTRODUCTION

Recently published tuning of the PI controller for integrator with dead time [7], [8]

$$F(s) = \frac{K_s}{s} e^{-T_d s} \tag{1}$$

based on triple real dominant pole showed to give excellent results. Based on the setpoint weighting with the control algorithm

$$U(s) = K_R\left[bW(s) - Y(s)\right] + \frac{K_p}{sT_i}\left[W(s) - Y(s)\right] \tag{2}$$

that can be shown to be equivalent to using prefilter

$$F_p(s) = \frac{bT_i s + 1}{T_i s + 1} \tag{3}$$

with T_i being the integral time constant, the method is based on solving the closed loop characteristic equation for a triple pole s_0 that for

$$\begin{aligned} A(s) &= s^2 T_i e^{T_d s} + K_R K_s (T_i s + 1) \\ \dot{A}(s) &= 2sT_i e^{T_d s} + s^2 T_d T_i e^{T_d s} + K_R K_s T_i \\ \ddot{A}(s) &= 2T_i e^{T_d s} + 4sT_d T_i e^{T_d s} + s^2 T_d^2 T_i e^{T_d s} \end{aligned} \tag{4}$$

requires to fulfill

$$A(s_0) = 0\ ;\ \dot{A}(s_0) = 0\ ;\ \ddot{A}(s) = 0 \tag{5}$$

Solution of the last equation corresponding to stability conditions yields

$$s_0 = -\left(2 - \sqrt{2}\right)/T_d \tag{6}$$

From the first two equations it follows

$$\begin{aligned} K_R &= 2\left(\sqrt{2} - 1\right)e^{\sqrt{2}-2}/\left(K_s T_d\right) \approx 0.461/\left(K_s T_d\right) \\ T_i &= \left(2\sqrt{2} + 3\right)T_d \approx 5.828 T_d \end{aligned} \tag{7}$$

By the requirement to cancel zero of the closed loop transfer function

$$F_{wy}(s) = \frac{K_s K_R (T_i s + 1)}{s^2 + K_R K_s (T_i s + 1) e^{-T_d s}} e^{-T_d s} \tag{8}$$

one get the prefilter denominator in (3) that removes overshooting typical for one degree of freedom PI controllers. Simultaneously, be requiring to cancel one of the triple real pole (5) by the prefilter numerator (3) that further accelerates the transient responses, one gets the setpoint weighting coefficient

$$b = \frac{1/|s_0|}{T_i} = \frac{2 - \sqrt{2}}{2} = 0.293 \tag{9}$$

In this way, very fast and smooth responses are achieved both in the regulatory as well as tracking control tasks.

2 WHY DO WE NEED FURTHER RESEARCH?

There are, however, several reasons to take the above result carefully.

Firstly, how do we know that this result gives optimal performance? To show, why this is a relevant question, let us consider P controller for the single integrator with dead time (1) tuned according to the double real dominant pole, when starting with the characteristic equation

$$A(s) = se^{T_d s} + K_R K_s \tag{10}$$

one gets in a similar way as above

$$\dot{A}(s) = e^{T_d s} + sT_d e^{T_d s} \tag{11}$$

So, the optimal tuning should be

$$\begin{aligned} s_0 &= -1/T_d \\ K_R &= 1/(\exp(1)T_d K_s) \approx 0.3679/(T_d K_s) \end{aligned} \tag{12a}$$

By simulation it is, however, easy to check that the loop remains monotonic, no overshooting and with one-pulse control up to

$$K_R = 0.3701/(\mathrm{T_d} K_s) \tag{12b}$$

Tuning (12b) gives shorter settling time and smaller values of IAE (Integral of Absolute Error), or of ISE (Integral of Squared Error) than (12a). Despite to the fact that (12b) represents relatively good approximation of (12a) a question arises, what is the really best tuning of the PI controller in terms of the settling time, IAE, ISE, etc.

The 2nd principal disadvantage of the proposed solution is that it does not consider the every time present control signal constraints.

The third principal disadvantage of the proposed design is that it guarantees excellent properties just in a single point. Since the existing plant have only seldom properties that can be characterized by fixed set of parameters, in practical applications it is important to keep specified technological properties, as e.g. non-overshooting control, monotonic output transients, or monotonic output transients with one smooth pulse of control having after a step change of input just one extreme point that guarantees low amplitudes of higher order harmonics in the control loop. These properties are, however, just rarely in focus of control research that (dominated by the mathematical convenience) concentrates mostly on performance criteria like gain margin, phase margin, maximum sensitivity, H_∞ norm, ISE, etc.

3 DISTURBANCE OBSERVER BASED PI$_1$ CONTROLLER

In the sense of above comments we have shown [4] that when using P controller extended by DOB based I action (by reconstructing and compensating piecewise constant disturbances at the plant input) it is possible to achieve several functional properties equivalent to those achieved usually by the PI controller with setpoint weighting. But, furthermore, also several new properties are achieved, as e.g. no windup effect, or simple and transparent tuning – similarly as in the case of series (interacting) PI controllers [1]. In this paper, this solution will be denoted as the PI$_1$ controller according to the order of the DOB filter.

One of the main advantages of the DOB based solution is given by the possibility to find experimentally a relatively simple mapping of regions of the two controller tuning parameters that guarantee one of the above mentioned technological performance criteria (non-overshooting, output monotonicity, or one pulse control). The one-pulse of control region is subset of monotonicity region that is subset of non-overshooting region included within the stability region. For $y(0) = 0$ it is identified by checking output monotonicity conditioned by validity of relations

$$0 \le y(t_1)/w \le y(t_2)/w, \forall 0 \le t_1 < t_2 \tag{13a}$$

and one pulse of control as

$$\begin{aligned} &sign(\dot{u}(t_1))sign(u(t_m)) \ge 0, \forall t_1 \in \langle 0, t_m \rangle \cup \\ &\cup\, sign(\dot{u}(t_2))sign(u(t_m)) \le 0, \forall t_2 \in \langle t_m, t_{sim} \rangle \end{aligned} \tag{13b}$$

whereby $u(t_m)$ corresponds to the maximal control signal amplitude during transient and t_{sim} represents simulation time that should be large than possible settling time.

Example of such a region of one-pulse control achieved experimentally (by simulation) for the controller in Fig. 1 with tuning parameter

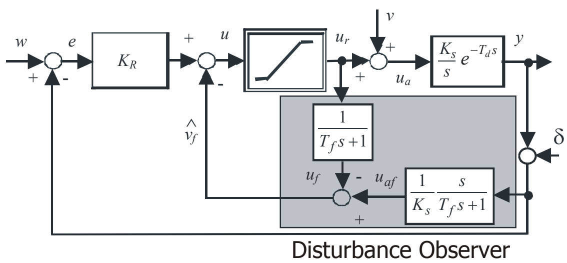

Fig. 1 DOB based PI$_1$ controller

$$\Omega_c = K_R K_s T_d \tag{14}$$

specifying the P controller gain and

$$\Omega_f = T_d / T_f \tag{15}$$

specifying dynamics of the reconstruction filter is given in Fig.2.

For the setpoint response the minimal values of IAE, or ISE of the one-pulse area correspond to the maximal value of Ω_c (i.e. to the maximal P controller gain) achieved for pure P-control and by increasing Ω_f (and decreasing Ω_c values) they increase.

For the disturbance response the PI_1 controller tuning guaranteeing one-pulse control and corresponding to minimal values of IAE is roughly given as

$$\Omega_f = 0.3\ ;\ \Omega_c = 0.2385\ ; \quad IAE_{vmin} = 13.4710\ ; \quad IAE_w = 4.1193 \qquad (16)$$

The ISE optimal tuning is given as

$$\Omega_f = 0.4\ ;\ \Omega_c = 0.1832\ ; \quad ISE_{vmin} = 14.2784\ ; \quad ISE_w = 2.9447 \qquad (17)$$

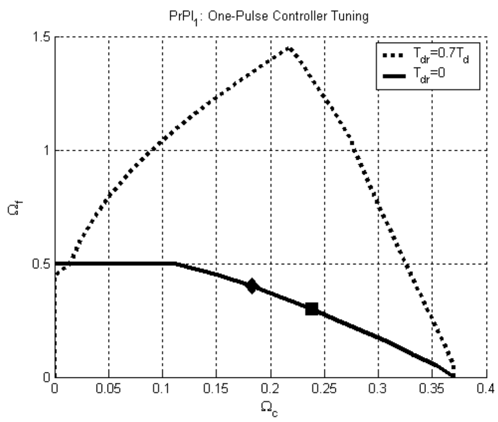

Fig. 2 Region of one-pulse of control of the PI_1 controller ($T_{dr} = 0$) tuning outlined by the full curve and both axes: ■ - tuning (16) giving minimal IAE value for disturbance response, ♦ - tuning (17) giving minimal ISE value for disturbance response; dotted – region of one pulse control for the predictive controller with the compensated time $T_{dr} = 0.7T_d$

When comparing these values with values corresponding to the PI controller for the triple real pole (7), (9)

$$IAE_w = 4.1204\ ; \quad ISE_w = 2.848\ ; \quad IEA_v = 12.6421\ ; \quad ISE_v = 17.657 \qquad (18)$$

it is to see that both controllers give similar quality - PI controller gives smaller values of IAE_v and ISE_w, but it is worse in the two complementary values IAE_w and ISE_v. Despite to this, the PI controller (6-9) seems to be more attractive by elegancy of its analytical derivation. For the PI_1 controller derivation of the optimal tuning based on the triple real pole [3] leads to complex controller tuning. Although its approximations by real values give responses that are close to the experimentally found optimum, elegancy of such derivation is lost. The PI_1 controller that is very close to the "series implementation" used in many commercial industrial PI controllers simply-cannot cover all parameter configurations of the parallel non-interacting controller [1].

3 ROBUST CONTROLLER DESIGN

When considering robust design, the plant parameters may be given over some interval, e.g. as

$$K_s \in \langle 0.9, 1.1 \rangle\ ;\ T_d \in \langle 1.0, 1.3 \rangle \qquad (19)$$

When the PI controller (7-9) is tuned at the point

$$K_{s\min} = 0.9;\ T_{d\min} = 1.0 \qquad (20)$$

but real parameters take values

$$K_{s\max} = 1.1\ ;\ T_{d\max} = 1.3 \qquad (21)$$

the resulting transients shown in Fig. 3 become oscillatory.

Let the choice of parameters (14-15) of the PI_1 controller be done in such a way that the whole uncertainty box with vertices crated by extreme values of the products $K_s K_R T_d$ and T_d / T_f is located within the one-pulse region. Due to its shape, the optimal setpoint response requiring maximal possible values of Ω_c and the optimal disturbance response corresponding to (16) or (17), the upper right corner of the uncertainty box corresponding to the maximal values of the plant parameters (21) should be placed e.g. to (16) that yields controller parameters

$$K_R = \frac{0.2385}{K_{s\max} T_{d\max}} = 0.1668 \quad T_f = \frac{1.3}{0.3} = 4.3333 \qquad (22)$$

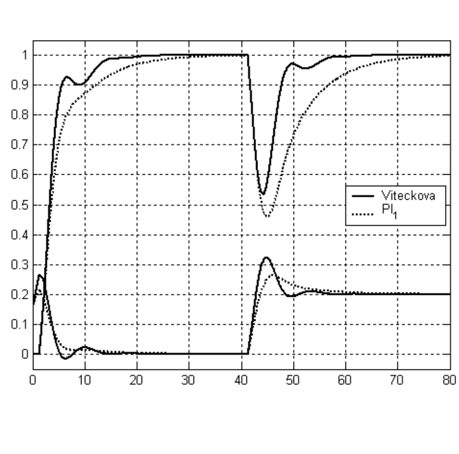

Fig.3 Transient responses of the PI controller tuned at the point (20) and the PI_1 controller responses with tuning (22) for the actual plant parameters (21)

Of course, one could say that it would be better to choose also for the PI controller tuning based on the maximal values of plant parameters (20). But, without knowing one-pulse region of characteristic parameters of the PI controller it is not possible to prove that the required properties will be guaranteed over the whole region of parameter changes. For the PI_1 controller

- the region of one-pulse control (Fig. 2) can be determined by very simple and numerically relatively robust and reliable program based on conditions (13),
- it is easy to check if the uncertainty box is contained in the above region.

For the PI controller computation of such areas will be much more sophisticated and numerically sensitive, because computing of the weighting coefficient b for (3) requires computing roots of the transcendental characteristic equation (4), for which it is not possible to use simple methods as e.g. the Newton-Raphson one. Up to now analytical computations of such task are possible by means of the Lambert W functions (see e.g. [2]) just for characteristic equations of the form

$$A(s) = (as + b)e^{cs} + d \tag{23}$$

Success of such approach will also depend on the shape of obtained regions – if they allow effective positioning of the uncertainty box.

4 DEADTIME COMPENSATION IN DOB

One of the strong principal advantages of the DOB based PI_1 controller is given by very easy dead time compensation in DOB. The disturbance reconstruction is based on estimating the plant input by means of the filtered inverse plant dynamics and comparing it with the equally filtered controller output. Precision of this comparison strongly depends on the phase shift of both channels disrupted by the plant dead time. It can be shown that this control structure is very similar to the "predictive" or "dead time" PI controller introduced by Shinskey [6].

The reconstruction seems to function ideally, when both channels are equally balanced by delays, i.e. after introducing into the reconstruction path leading from the controller output (Fig.4) the same delay as it is in the loop with the plant. In such a situation, without an input disturbance v the DOB should give zero output and so it seems that the whole loop can be tuned as simple P controller having analytically designed optimal gain (12a), or the experimentally achieved one (12b).

Since in the robust design one has to consider also possible plant/model mismatch, the curves outlining required performance areas (e.g. the one-pulse control area outlined in Fig.2 for $T_{dr} = 0.7T_d$) have to be draw for all possible situations with $T_{dr} \neq T_d$, or at least, for the limit situations. One obtained region enables just nominal controller tuning. In such a case with exactly know process parameters the one-pulse performance will be achieved for any working point with $T_{dr} = 0.7T_d$ and (Ω_c, Ω_f) chosen within the dotted area in Fig. 2. When e.g. choosing points $(0.35, 0.2)$ and $(0.32, 0.5)$, in comparing with the "optimal" PI controller the resulting transient responses shown in Fig. 5. may either have reasonably improved setpoint respons, or disturbance response.

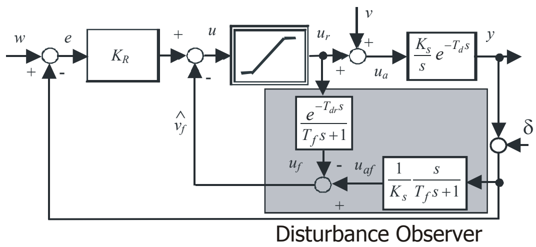

Fig. 4 Dead time compensation in DOB

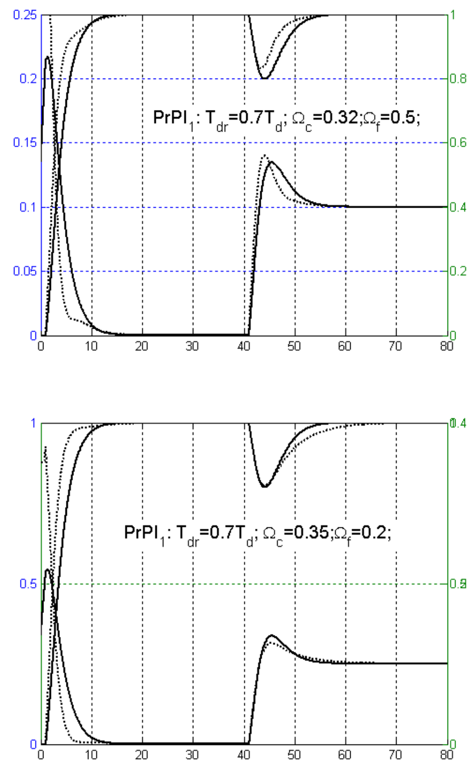

Fig. 5 Transient responses corresponding to the PI controller (7-9) (full) and to the predictive controller (Fig. 4) for two different tuning of the one-pulse control (dot)

Achieved transients are much less oscillatory than e.g. those achieved by optimizing only the ISE performance index [5]

As already mentioned, for robust control it would be required to get the one-pulse areas for all considered ratios T_{dr}/T_d and to place the uncertainty box in such a way enabling being included in all of them.

5 CONCLUSIONS

Numerically simple and very effective approach for designing robust PI and predictive (dead time) PI controllers for first order systems with dead time was proposed that is based on determining areas of controller parameters corresponding to chosen performance specification:

The method is easier applicable to the DOB based controllers, since it does not require the numerically sensitive determination of real roots of the transcendent characteristic equation.

We plan to implement the method in form of internet application and to make it available to public, soon.

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6 REFERENCES

[1] Åström, K. J., Hägglund, T.: PID controllers: Theory, design, and tuning – 2nd ed., Instrument Society of America, Research Triangle Park, NC, 1995.

[2] Chyi Hwang and Yi-Cheng Cheng: Use of Lambert W Function to Stability Analysis of Time-Delay Systems, 2005 American Control Conference June 8-10, 2005. Portland, OR, USA, FrB09.6, 4283- 4288

[3] Huba,M., Bisták,P., Skachová,Z., Žáková,K.: P- and PD-Controllers for I1 and I2 Models With Dead Time. 6th IEEE Mediterranean Conference on Control and Systems, Alghero 1998, 514-519.

[4] Huba, M.: Disturbance observer based controller for integrator with dead time. Submitted to *IEEE Trans. Ind. Electr*.

[5] Leu J.-F.; Tsay S.-Y.; Hwang Ch.: Optimal tuning of PID-deadtime controllers for integrating and unstable time-delayed processes. Journal of Chinese Institute of Engineers, Taipei, Taiwan, 2006, vol. 29, n°2, pp. 229-239

[6] Shinskey, F.G.: PID-Deadtime Control of Distributed Processes. *IFAC Workshop on Digital Control: Past, present and future of PID Control*, Terassa, Spain 2000, 14-18.

[7] M. Vítečková and A. Víteček: Two-degree of freedom controller tuning for integral plus time delay plants. Int. Conference Cybernetics and Informatics, February 10 - 14, 2008, ŽDIAR, Slovak Republic

[8] Vítečková, M., Víteček, A.: Two-degree of Freedom Controller Tuning for Integral Plus Time Delay Plants. *ICIC Express Letters. An International Journal of Research and Surveys*. Volume 2, Number 3, September 2008, Japan , pp. 225-229.