Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation, and Mathematics

PROCEEDINGS

of the 18$^{\mathrm{th}}$ International Conference on Process Control
Hotel Titris, Tatranská Lomnica, Slovakia, June 14 – 17, 2011
ISBN 978-80-227-3517-9
http://www.kirp.chtf.stuba.sk/pc11

Editors: M. Fikar and M. Kvasnica

Mareš, J., Doležel, P., Dušek, F., Procházka, A.: Reactor Furnace Control – PID and Predictive Methods Comparison, Editors: Fikar, M., Kvasnica, M., In *Proceedings of the 18th International Conference on Process Control*, Tatranská Lomnica, Slovakia, 243–246, 2011.

Full paper online: http://www.kirp.chtf.stuba.sk/pc11/data/abstracts/022.html

Reactor Furnace Control - PID and Predictive Methods Comparison

J. Mareš*, P. Doležel, F.Dušek**, A. Procházka***

*Institute of chemical technology, Department of computing and control engineering
Technická 5, 166 28 Prague 6, Czech Republic
e-mail: jan.mares@vscht.cz,uprt.vscht.cz
**University of Pardubice, Faculty of electrical engineering and informatics, Department of process control,
Nám. Čs. Legií 565, 532 10 Pardubice, Czech Republic
e-mail: petr.dolezel@upce.cz,www.upce.cz

Abstract: Paper deals with different techniques of nonlinear reactor furnace control. The first part briefly describes the real system (reactor furnace), which is a nonlinear system because of different heat transport mechanisms. Then different approaches to the system control are described. Firstly standard technique using PID controller, and secondly two predictive control strategies (Generalized Predictive Controller and Neural Network Predictive Controller).

1. INTRODUCTION

Different techniques of the reactor furnace control are described and compared in the paper.

Furnace is made for chemical reactor heating. The reactor provides measurements of oxidation and reduction qualities of catalyzers in the different temperatures. It is necessary to consider a nonlinear furnace behavior, because of huge range of reactor temperature (Dušek, et all.,1997).

2. REACTOR FURNACE DESCRIPTION

The furnace base is a cored cylinder made of insulative material. On the inner surface there are two heating spirals. Spirals are powered by the voltage 230 V. In the middle of the cylinder there is a reactor. The reactor temperature is measured by one platinum thermometer (see Figure 1).

The system is a thermal process with two inputs (spiral power and ambient temperature) and one output (reactor temperature). Thus, controlled variable is the reactor temperature and manipulated variable is spiral power, ambient temperature is measured error.

Nonlinearity of the system is caused by heat transfer mechanism.

When the temperature is low, heat transfer is provided only by conduction. However, when the temperature is high, radiation presents an important transfer principle.

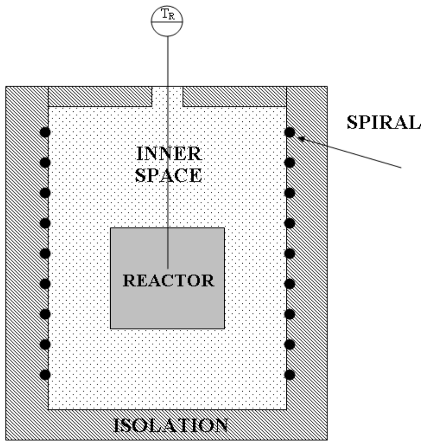

Fig. 1. Reactor furnace chart

Nonlinear mathematical model (set of four differential equations) and its linearization is described in (Mareš et all., 2009) and (Mareš et all., 2010a).

3. PID CONTROL

The first approach how to control the reactor furnace is the simplest way – PID control, where gain and time constants were set according to T_{Σ} method, more in (Kuhn, 1995). The method gives the PID control response slow but very robustness. Even nonlinear systems are possible to control quite satisfactorily.

The only necessity for the controller parameters estimation is to measure the step response of the system. Then we can calculate gain and parameter T_{Σ}, see figure 2 and equation (1). Constants of the controller are calculated from these parameters, according to table 1.

$$\int_0^{T_\Sigma} y(t)dt = \int_{T_\Sigma}^{\infty} [y(\infty) - y(t)]dt \qquad (1)$$

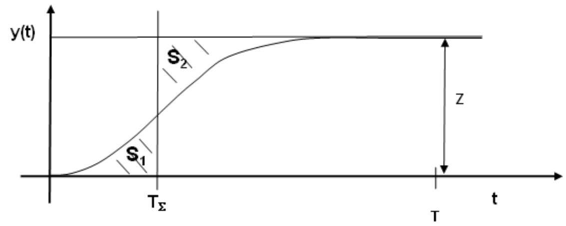

Fig. 2. T_Σ method principle

Table 1 – PID parameters calculation

	r_0	T_I	T_D
PI	0,5/Z	$0,5.T_\Sigma$	0
PID	1/Z	$0,66.T_\Sigma$	$0,167.T_\Sigma$

The step response was measured (step of the spiral power 0 – 100 W) and the PI controller parameters were estimated, table 2.

Table 2 – PI controller setting

	r_0	T_I	T_D
PI	3,38	223	0

The control experiment was realized at the system. Results are shown in figure 3, where the first chart shows the manipulated variable, the second chart shows the set point and controlled variable and the third chart shows the error between set point and controlled variable.

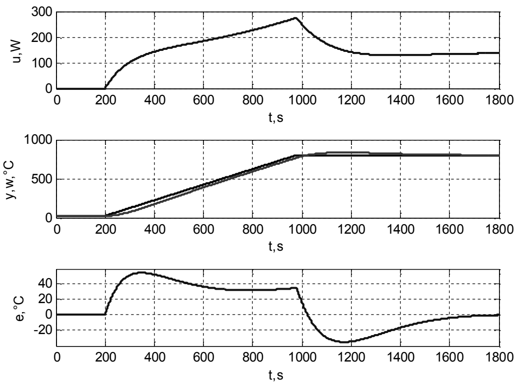

Fig. 3.PID control

4. GENERALIZED PREDICTIVE CONTROL

Generalized Predictive Control (GPC) belongs to the group of complex predictive controllers where model is needed. We assume the model in the form of equation (2).

$$A(z^{-1})y(k) = B(z^{-1})u(k-1) + C(z^{-1})\frac{e(k)}{\Delta} \tag{2}$$

where *A, B, C* are polynomials, *y (k)* is model output, *u (k)* is model input *e (k)* is output error and Δ is described by $\Delta = 1 - z^{-1}$. It is possible to convert (2) to the form of equation (3)

$$\overline{A}(z^{-1})y(t) = B(z^{-1})\Delta u(t-1) + C(z^{-1})e(t) \tag{3}$$

where $\overline{A} = \Delta.A$.

The model is used for the calculation of the future output prediction. There are several different methods how to calculate it. One of the simplest ways (using the inverse matrix) is described in this chapter.

The prediction of N steps is possible to rewrite by the set of equations (4).

$$\begin{aligned}
& y(k+1) = b_1\Delta u(k) + b_2\Delta u(k-1) + \dots \\
& \dots + b_{n+1}\Delta u(k-n) - a_1 y(k) - a_2 y(k-1) - \dots - a_{m+1} y(k-m) \\
& y(k+2) = b_1\Delta u(k+1) + b_2\Delta u(k) + \dots \\
& \dots + b_{n+1}\Delta u(k-n+1) - a_1 y(k+1) - a_2 y(k) - \dots - a_{m+1} y(k-m+1) \\
& y(k+3) = b_1\Delta u(k+2) + b_2\Delta u(k+1) + \dots \\
& \dots + b_{n+1}\Delta u(k-n+2) - a_1 y(k+2) - a_2 y(k+1) - \cdots \\
& \qquad .. - a_{m+1} y(k-m+2) \\
& \vdots \\
& y(k+N) = b_1\Delta u(k+N-1) + b_2\Delta u(k+N) + \dots \\
& \dots + b_{n+1}\Delta u(k+N-n+1) - a_1 y(k+N-1) - \\
& \qquad - a_2 y(k+N) - \dots - a_{m+1} y(k+N-m+1)
\end{aligned} \tag{4}$$

In matrix form it is possible to write

$$\mathbf{A}.\begin{bmatrix} y(t+1) \\ y(t+2) \\ \vdots \\ y(t+N) \end{bmatrix} = \mathbf{B}.\begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \vdots \\ \Delta u(t+N-1) \end{bmatrix} + \widetilde{\mathbf{B}}.\begin{bmatrix} \Delta u(t-1) \\ \Delta u(t-2) \\ \vdots \\ \Delta u(t-n) \end{bmatrix} + \widetilde{\mathbf{A}}\begin{bmatrix} y(t) \\ y(t-1) \\ \vdots \\ y(t-m) \end{bmatrix} \tag{5}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -a_N & -a_{N-1} & \cdots & 1 \end{bmatrix} N \text{x} N;\ \mathbf{B} = \begin{bmatrix} b_1 & 0 & \cdots & 0 \\ b_2 & b_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ b_N & b_{N-1} & \cdots & b_1 \end{bmatrix} N \text{x} N$$

$$\widetilde{\mathbf{A}} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{m+1} \\ a_2 & \cdots & a_{m+1} & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \end{bmatrix} N \text{x}(m+1);\ \widetilde{\mathbf{B}} = \begin{bmatrix} b_2 & b_3 & \cdots & b_{n+1} \\ b_3 & \cdots & b_{n+1} & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \end{bmatrix} N \text{x} n$$

Future output prediction of the system *y(t+i)* is possible to calculate by multiplying the equation (5) by the inverse matrix $\mathbf{A}^{-1}$, equation (6).

$$\begin{bmatrix} y(t+1) \\ y(t+2) \\ \vdots \\ y(t+N) \end{bmatrix} = \mathbf{A}^{-1}.\mathbf{B}.\begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \vdots \\ \Delta u(t+N-1) \end{bmatrix} + \mathbf{A}^{-1}.\widetilde{\mathbf{B}}.\begin{bmatrix} \Delta u(t-1) \\ \Delta u(t-2) \\ \vdots \\ \Delta u(t-n) \end{bmatrix} + \mathbf{A}^{-1}.\widetilde{\mathbf{A}}\begin{bmatrix} y(t) \\ y(t-1) \\ \vdots \\ y(t-m) \end{bmatrix} \tag{6}$$

Last two terms describes only the system history, therefore it is possible to put them together to the matrix **F** and the vector of historical output and inputs $\mathbf{h} = [\mathbf{y} \quad \mathbf{u}]^T$. Thus, the equation of prediction is possible to write in the form of equation (7).

$$\mathbf{y} = \mathbf{G}.\mathbf{u} + \mathbf{F}.\mathbf{h} \tag{7}$$

The aim of GPC is to calculate the vector of manipulated variable by minimizing of the cost function (8).

$$J = \mathbf{e}_N^T.\mathbf{e}_N + \lambda.\mathbf{u}^T.\mathbf{u} \quad (8)$$

where **e** is vector of control errors (length N), **u** is vector of manipulated variables (length N) and λ is weighting coefficient.

The cost function can be modified using output prediction (9) and set point vector **w.**

$$J = (\mathbf{w} - \mathbf{G.u} + \mathbf{F.h})^T (\mathbf{w} - \mathbf{G.u} + \mathbf{F.h}) + \lambda.\mathbf{u}^\mathbf{T}.\mathbf{u} \quad (9)$$

We can calculate the vector of manipulated variable **u** analytically using the square norm. Then we get equation (10).

$$\mathbf{u} = (\mathbf{G}^\mathbf{T}.\mathbf{G} + \lambda.\mathbf{I})^{-1}.\mathbf{G}^\mathbf{T}.(\mathbf{w} - \mathbf{F.h}) \quad (10)$$

We usually need only one actual value of the manipulated variable (the first element of the vector) therefore the final form of the control law is equation (11).

$$\Delta u = \mathbf{K}\left\{.\begin{bmatrix} w(t) \\ w(t+1) \\ \vdots \\ w(t+N) \end{bmatrix} - \mathbf{F}.\begin{bmatrix} \Delta u(t-1) \\ \Delta u(t-2) \\ \vdots \\ \Delta u(t-n) \\ y(t) \\ y(t-1) \\ \vdots \\ y(t-m) \end{bmatrix}\right\} \quad (11)$$

where **K** is the first row of matrix $(\mathbf{G}^\mathbf{T}.\mathbf{G}+\lambda.\mathbf{I})^{-1}.\mathbf{G}^\mathbf{T}$

GPC theory is formulated for the group of linear systems control but in the case of nonlinear systems it is not possible to use it because the linear model is not able to describe the nonlinear process well. Nonlinear system control needs nonlinear model or linearized model (this case).

In the case of piecewise linearized GPC we can do the linearization of the model and formulate it in the form of (3). Matrices **G** and **F** are possible to calculate from this form in defined number of linearization points, equation (7). Thus, the controller will switch between pre-calculated setting during control experiment (according to reactor temperature). Moreover, it is possible to interpolate between two adjoining settings. Nonlinear behavior of the system is substituted by piecewise linearized model. Complex description of this approach is in (Mareš et all.,2010b).

The control experiment was realized too. Results are shown in figure 4, where the description of charts is the same as in previous example.

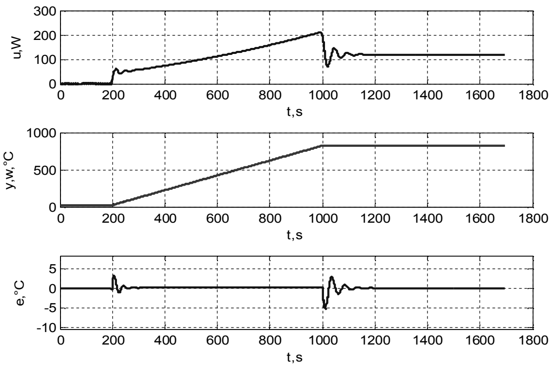

Fig. 4.Linearized GPC

5. NEURAL NETWORK PREDICTIVE CONTROL

Another approach to predictive control is described in this section. Predictive controller here uses a neural network (NN) model of nonlinear plant to predict future plant performance. The controller then calculates the control input that will optimize plant performance over a specified future time horizon.

The first stage of NN predictive control is to design a neural network which represents the dynamics of the plant. The prediction error between the plant output and NN output is used as the neural network training signal (see figure 5).

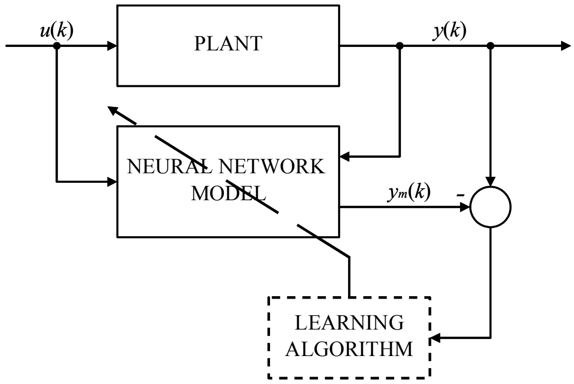

Fig. 5. – NN model identification

This neural network can be trained offline in batch mode, using data collected from some experiments with the plant. Any backpropagation algorithm can be used for network training. Process of neural network model design is discussed in detail in (Taufer et all., 2008).

In this control technique, neural network predicts the plant response over a specified time horizon. The predictions are used by some search technique to determine the control signal that minimizes the following performance criterion over the specified horizon *N*

$$J = \mathbf{e}_N^T.\mathbf{e}_N + \lambda.\mathbf{u}^T.\mathbf{u} \quad (12)$$

where $\mathbf{e}_N$ and **u** are the same vector as in (8).

The figure 6 illustrates the NN model predictive control process. The controller consists of the neural network plant model and the optimization block. The optimization block determines the values of $u'(k)$ that minimize the criterion J, and the optimal $u'(k)$ is input to the plant.

It is obvious, that key part of block diagram below is optimization block or used search technique, more precisely. Mostly, optimal $u'(k)$ is not found every sample time, because only fixed number of iterations is performed per one sample time.

Whole control technique is included in Neural Network Toolbox of Matlab.

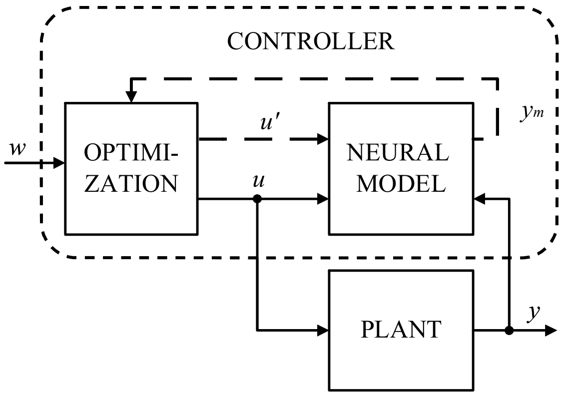

Fig. 6. NN predictive control

Control experiment with NN predictive controller was performed (Prediction horizon N = 20, λ = 0.1, golden section search routine). Neural network model was trained offline with Levenberg-Marquardt training algorithm and its topology is illustrated in figure 7. Control performance can be found in figure 8.

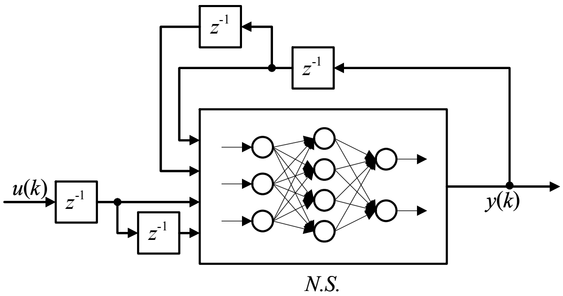

Fig. 7. NN model

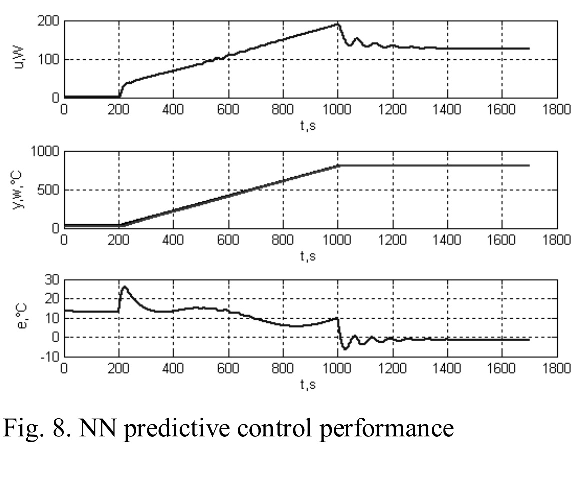

Fig. 8. NN predictive control performance

6. CONCLUSIONS

Paper deals with different techniques of real system (reactor furnace) control. As the introduction there is a brief description of the plant, which behavior is nonlinear because of the range of the reactor temperature.

The first part describes the simplest way – PID controller, where gain and time constants were set according to T_{Σ} method. The method gives the PID control response very slow but robustness. Therefore, nonlinear systems are possible to be controlled quite satisfactorily.

Then, the second part describes the predictive control design which uses firstly linearized mathematical model and secondly neural network model.

As conclusion, it is possible to say, that all three approaches gives satisfactory results and are able to control nonlinear system properly.

ACKNOWLEDGMENT

The work has been supported by the funds No. MSM 6046137306 and No. MSM 0021627505 of Ministry of Education of the Czech Republic. This support is very gratefully acknowledged.

7. REFERENCES

DUŠEK, F., ČIČMANEC, P., BULÁNEK, R.: *Řízení pece na zadaný průběh*In: 11th Conference PROCESS CONTROL '97, Tatranské Matliare 8.-11.6.1997, Volume 2,s.

KUHN, U. *A practical tuning rule for PID controllers: the t-sum design*. Automatisierungstechnische Praxis, 37(5), 10-16, 1995.

MAREŠ, J., DUŠEK, F., HONC, D. *Náhrada reálného zařízení modelem v Simulinku*. Sborník příspěvků z Technical Computing Prague 2009. Praha, KANAG – TISK, s.r.o. 69, 119 str. ISBN 978-80-7080-733-0.

MAREŠ, J., DUŠEK, F. *Nelineární a linearizovaný model reaktorové pece*. In *Proceedings of Conference ARTEP'10"*, 24.-26. 2 2010a.Technická univerzita Košice, 2010. Pp. 27-1 – 27-14. ISBN 978-80-553-0347-5.

MAREŠ, J., DUŠEK, F. *Reactor furnace predictive control*. In Proceedings of XXXVth Seminary ASR'10 „Instruments and Control", VSB- Technical University Ostrava, 2010b.

TAUFER, I.; DRÁBEK, O.; SEIDL, P. Umělé neuronové sítě – základy teorie a aplikace (10), *CHEMagazín*, 2008, vol. XVII, n. 1, p. 35-37. ISSN 1210-7409.