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# The Empirical Mode Decomposition in Real-Time

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**Abstract:** The paper devotes analysis of environmental time series by using the on-line empirical mode decomposition (OEMD). The environmental data were measured by meteorological stations which are deployed in the southern part of Czech Republic. The EMD algorithm was modified for the possibility of the on-line analysis of environmental time series.

## 1. INTRODUCTION

During processing of environmental data to consider the state of the ecosystem, the stationarity or periodicity of the measured data is usually assumed. In fact, the observed data reflect the characteristics of the ecosystem, which is generally nonlinear, stochastic and nonstationary. The results obtained, given the very simplistic assumptions might therefore lead to incorrect conclusions and to obtain distorted characteristics either in time or frequency domain. Since similar problems encountered at each analyzing nonstationary stochastic systems, the EMD (Empirical Mode Decomposition) algorithm, developed by N. E. Huang in 1998 for NASA (Huang, et al. 1998), attracted much attention. Huang combined EMD algorithm with the algorithm for the Hilbert spectral analysis and created the so-called Hilbert-Huang Transformation (HHT), which is applicable for analysis of nonlinear, stochastic and nonstationary processes.

## 2. EMPIRICAL MODE DECOMPOSITION

An EMD algorithm decomposes adaptively the signal $x(t)$ into intrinsic mode functions $c_i(t)$, $i = 1, 2, \dots, n$ and into residue $r(t)$:

$$x(t) = \sum_{i=1}^{n} c_i(t) + r(t), \qquad (1)$$

where $n$ means the number of IMF functions. Residue $r(t)$ reflects the average trend of a signal $x(t)$ or a constant value. Intrinsic mode functions (IMF) are signals with following characteristics:

In the whole dataset, the number of extremes (minima and maxima) and the number of zero-crossings must either equal or must differ by a maximum of one.

Each point, that is defined as mean value of envelopes defined by local maxima and local minima is zero.

The algorithm for searching of intrinsic mode functions is based on a procedure called "sifting", described e.g. in (Zhaohua 2009) and (Zhaohua 2010).

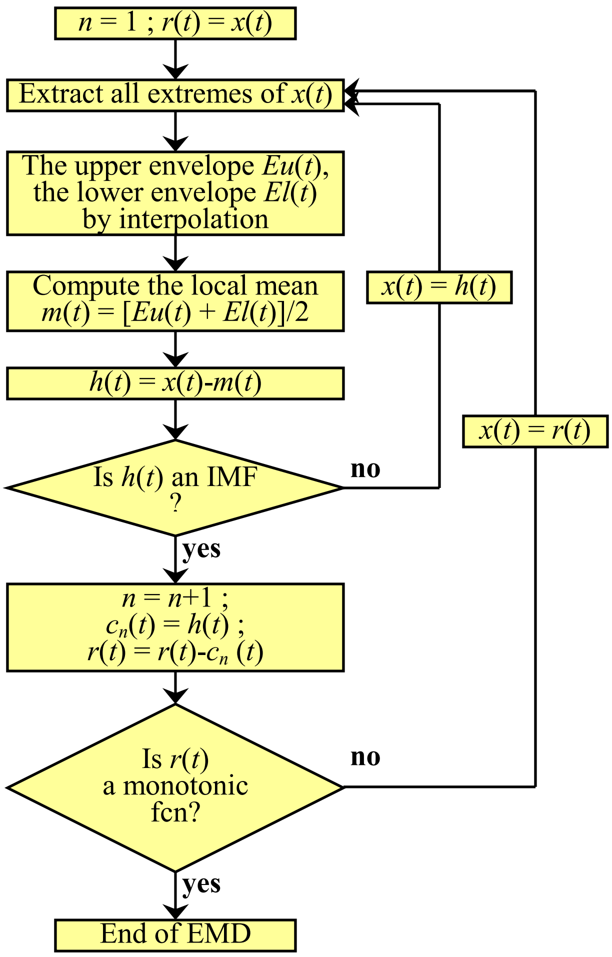



Fig 1.Flowchart of Empirical Mode Decomposition algorithm

The algorithm proceeds in the following steps (see Fig. 1):

1. Create upper envelope $E_u(t)$ by local maxima and lower envelope $E_l(t)$ by local minima of data $x(t)$.

2. Calculate the mean of upper and lower envelope
$$m_1(t) = \frac{E_{u1}(t) + E_{l1}(t)}{2}. \tag{2}$$

3. Subtract the mean from original data
$$h_1(t) = x(t) - m_1(t). \tag{3}$$

4. Verify that $h_1(t)$ satisfies conditions for IMFs. Repeat steps 1 to 4 with $h_1(t)$, until it is an IMF.

5. Get first IMF (after $k$ iterations)
$$c_1(t) = h_{1(k-1)}(t) - m_{1k}(t). \tag{4}$$

6. Calculate first residue
$$r_1(t) = x(t) - c_1(t) \tag{5}$$

7. Repeat whole algorithm with $r_1(t)$, $r_2(t)$, … until residue is monotonic function.

8. After $n$ iterations $x(t)$ is decomposed according to equation (1).

## 3. ON-LINE ANALYSIS

The algorithm described in chapter 2. is calculated off-line over the entire measured data range. Since the dataset can be very large, the sifting process can be time-consuming and computationally very demanding. Therefore, an algorithm that processes the data gradually, by moving time windows, was created. The advantage of floating time windows is mainly significant in accelerating of decomposition into IMF functions.

For off-line decomposition, the computation time is not so much restricted and the interval edges, that might be distorted, can be omitted from the analysis. But these two problems become serious, when the EMD algorithm for on-line analysis is used. The following procedure was used to overcome these problems:

- The floating time window is created. The window range:
$$\langle (t - T_w); t \rangle, \tag{6}$$
where $t$ is actual time point and a $T_w$ is length of window. The decomposition process is at any point of time evaluated only in the appropriate window range. So the time needed for compute IMFs does not grow with simulation time.

- To reduce distortion of decomposition at the end of measurement interval, the currently known courses of IMFs are used to estimate their future course and to estimate their future local extremes needed for EMD algorithm.

The size of the time window affects frequencies, which ones will be detected during sifting in the IMFs and which ones will be included within residue. Generally, the longer the window, the lower frequencies (and therefore the longer periods) will appear in IMFs.

The practical realization of algorithm is implemented in the software MATLAB and its simulation toolbox SIMULINK. As a basis for the algorithm is used modified Zhaohua MATLAB function „eemd()“ – see (Zhaohua 2010). The modification consists in replacing the interpolation functions for creating of envelopes with functions that allow also interpolation. The modified eemd() function is built into the s-function „s-emd()“. The main tasks of the s-function are:

- To create and to maintain moving time window during simulation.

- To maintain dynamic global variables of type one-, two- and three-dimensional array of variable length. These variables store the source data, residue and IMFs with their variances.

## 4. IMPLEMENTATION

Modified On-line Empirical Mode Decomposition (OEMD) algorithm described in chapter 3 is demonstrated on soil temperature measurements from the meteorological station by the environmental project TOKENELEK. (Fig. 2) shows the temperature $\vartheta$ during September 2010, the sampling period is 10 minutes.

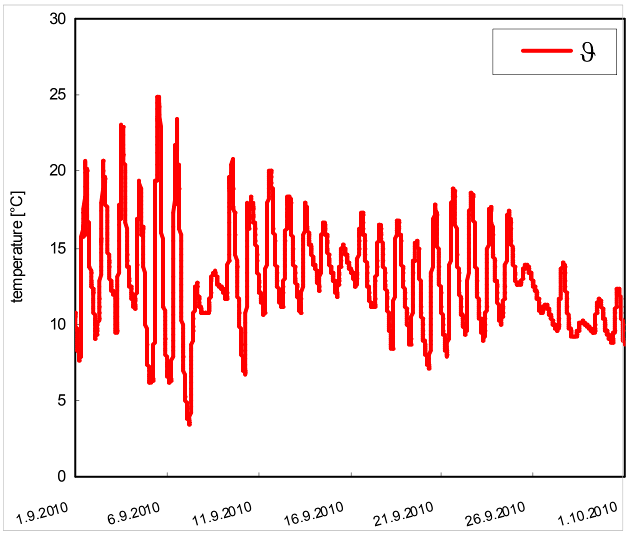
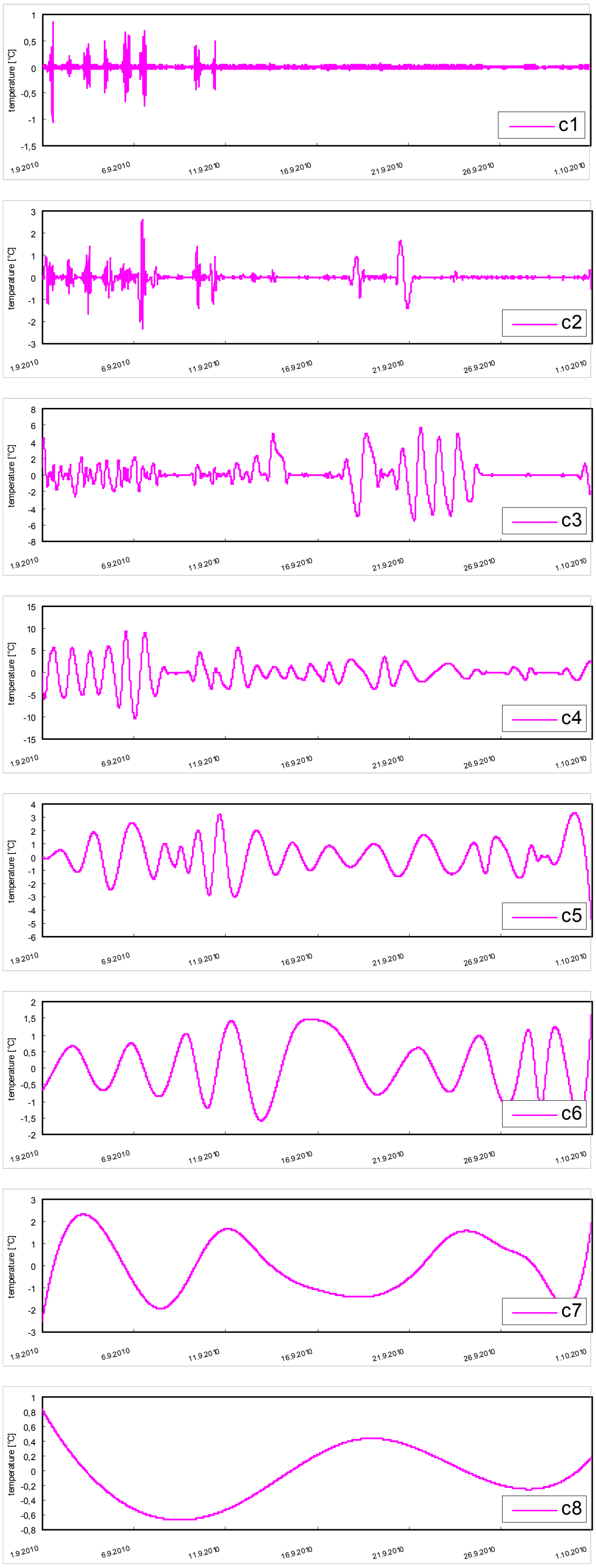



*Fig. 2 The time range of temperature $\vartheta$.*



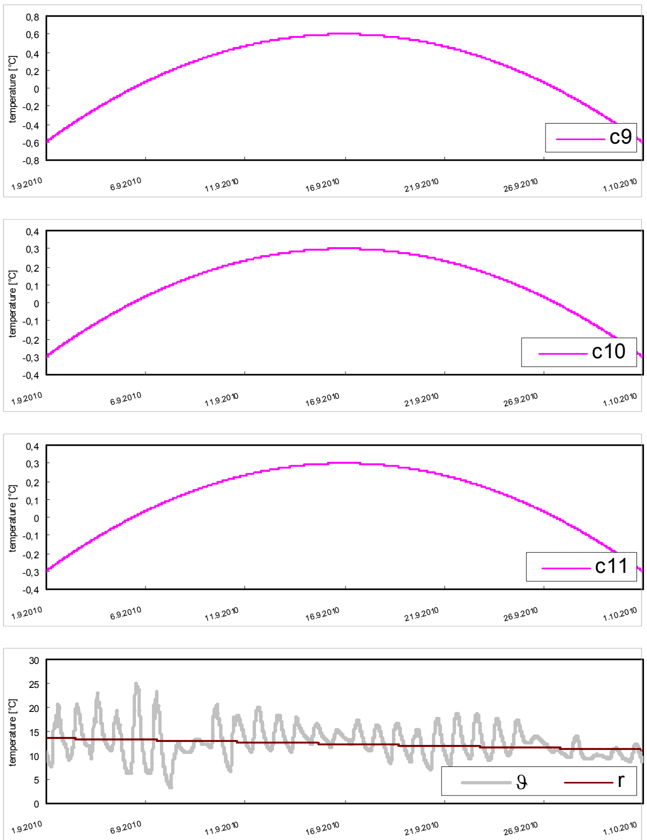




*Fig. 3 Intrinsic Mode Functions (IMF) and Residue obtained off-line once around a course. Pay attention to changing scale of vertical axis. The residue is compared to measured data $\vartheta$.*

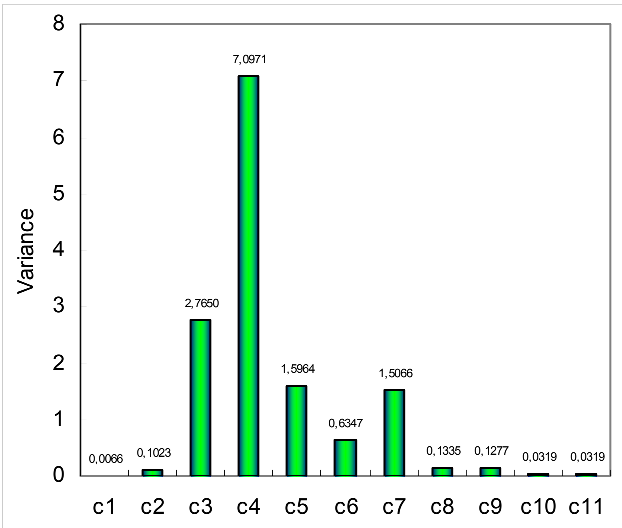



*Fig. 4 Comparison of variances of (off-line) IMF. The larger variance, the more important component it represents.*

The experimental analysis has been divided into two parts. An off-line EMD analysis was carried out in the first part. The resulting functions are shown in the graph (Fig. 3) and serve as reference samples of IMF functions. Variance was also found for every IMF. Variances were used as a simple benchmark to determine how significant component of the initial data each IMF represents. As shown in (Fig. 4), the most important component is $c_4$.

(Fig. 5) compares original data $\vartheta$ with filtered temperature $\vartheta_f$, that is declared by formula:

$$\vartheta_f(t) = \sum_{i=2}^{9} c_i(t) + r(t). \qquad (6)$$

The same source data (Fig. 2) has been processed by on-line EMD algorithm in the second part of the experiment. The size of the time window has been set at 1008, which given the sampling density represented 7 days.

The rectangle shown in graph (Fig. 6) represents the time window during the simulation. One set of IMF functions that has been generated inside the time window is shown in the graph (Fig. 7). It is obvious that the residue is more curved in comparison to off-line decomposition and also some IMFs have higher variance. This phenomenon is a necessary and expected consequence of a shorter time range of analyzed data.

Along with finding the IMF functions their variance were estimated. On the basis of the variances it was decided which IMFs will be included into partial filtered function $\vartheta_f(t)$.

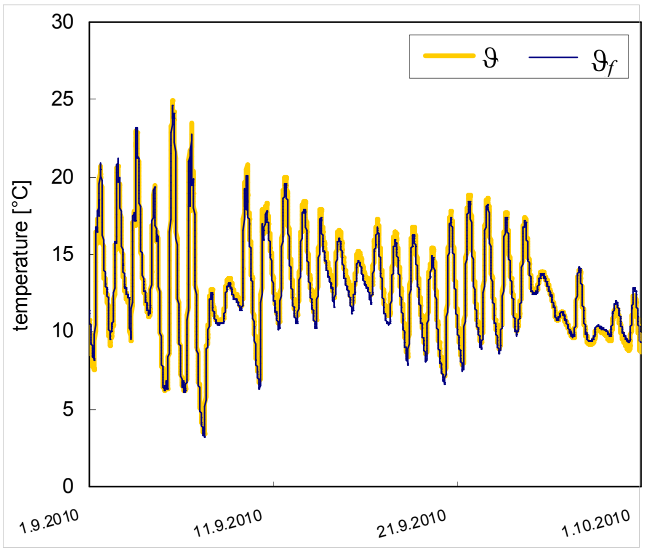



*Fig. 5 Comparison of temperature trends $\vartheta$ and $\vartheta_f$. The thick line represents the original measured data; the thin line represents the filtered function obtained by adding the selected major IMFs and residue.*

(Fig. 8) shows the course of several partial filtered functions compared to the original source data. Thick light line represents the source data; thin gray scaled lines show partial filtered functions from selected time windows.

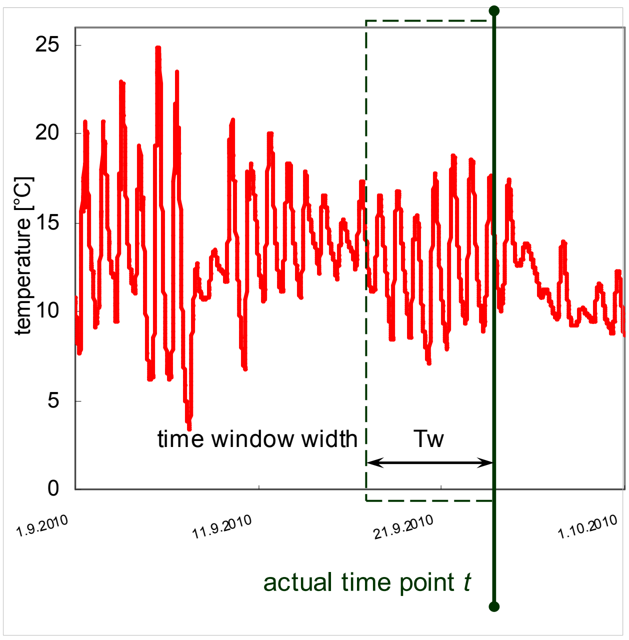
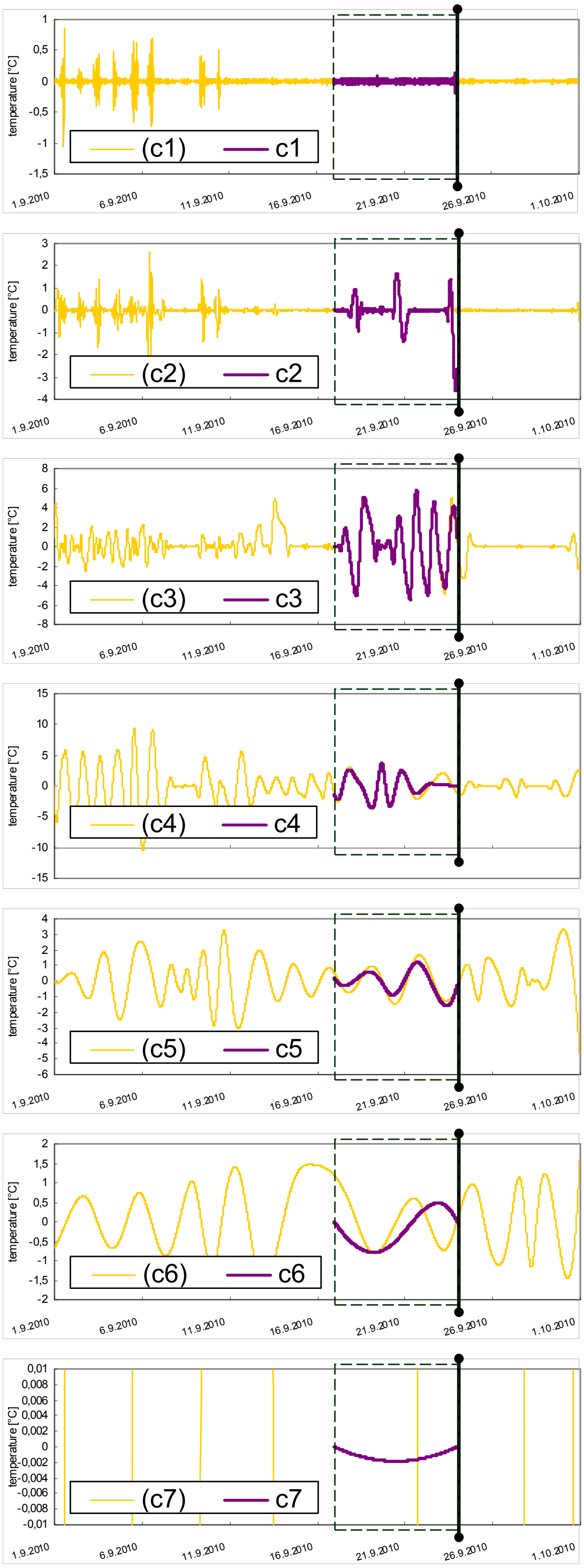



*Fig. 6 The symbolic representation of moving time window.*



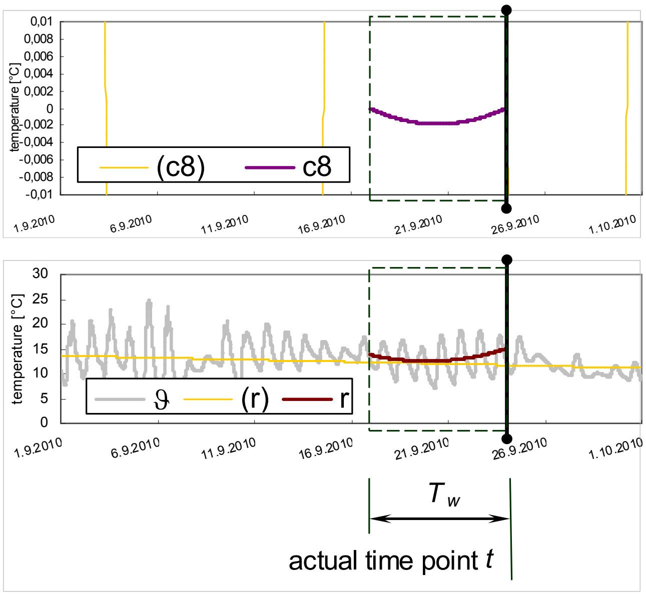




*Fig. 7 Comparison of on-line IMF obtained by on-line EMD analysis for one time point t and offline IMF (see Fig. 3). The number of on-line and off-line IMF differs, so the comparison mainly illustrates how sifting process depends on window size.*

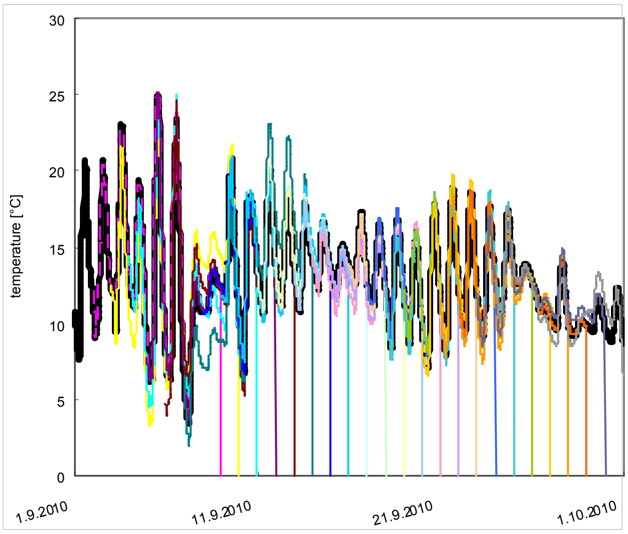



*Fig. 8 Comparison of measured temperature $\vartheta$ and (on-line) partial filtered functions $\vartheta_{fi}$. The measured temperature is represented by thick light line; the courses of filtered temperatures for selected time windows are represented by thin lines with varying greyscales. The intervals between displayed partial functions are 1 day; the length of time window is $T_w$=7 days.*

## 5. CONCLUSION

Empirical Mode Decomposition (EMD) is a progressive method that combines the signal analysis in time even in frequency domain. In field of environmental non-stationary data streams, almost repetitive sequences with very various periodicities often appear, but exact repetition of events occurs rarely. For this reason, the EMD analysis of these systems is very convenient. Real-time data processing provides results qualitatively similar to the offline analysis. Comparing of both methods shows that on-line analysis with a moving time window is much faster and significantly reduces the computational complexity. Off-line analysis on the contrary provides a slightly more detailed decomposition because it captures even very low frequencies. Data obtained from the experiments provided a number of suggestions for further work, with a wide range stems mainly from the short-term prediction of the internal mode functions for more precise decomposition in real time.

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