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Robust decentralized controller design for 3D crane

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Abstract: The subject of this paper is to design robust decentralized PID controllers for the 3Dcrane to stabilize motion of the cart along axes-x, axes-y using the Small Gain Theorem, and Parameter Dependent Lyapunov Functional (PDLF) in time domain. The obtained results were evaluated and verified in the Matlab simulink and on the real model of the 3DCrane.

Keywords: decentralized control, PID controller, Parameter Dependent Lyapunov Functional (PDLF)

1. INTRODUCTION

The industrial crane model 3DCrane is one of the real processes built for control education and research at Department of Information Engineering and Process Control. The 3DCrane is a nonlinear electromechanical MIMO system having a complex dynamic behavior and creating challenging control problems as nonlinear, interactions between subsystems corresponding to motion of cart along the axes-x and axes-y, length of the payload lift-line dynamic changed … The technical equipments allow us to realize control crane by classical and advanced control method (INTECO Ltd.: 3DCrane User's manual).

The main aim of paper is to use knowledge of multivariable (Multi Input and Multi Output) system and stabilization of decentralized control systems (Zhining et al. 1997), knowledge about robust control of linear systems in the frequency domain (Veselý et al. 2006) and in the time domain to design robust PID/PD decentralized controllers stabilizing the cart motion process of 3DCrane along the both axis x/y with the different length of the payload lift-line (robust stability). Furthermore, the resulting feedback control system with designed controllers must satisfy robust performance conditions for tracking the desired position of the cart.

2. OVERVIEW OF THE 3DCRANE SYSTEM WITH ARTIFICAL INTERACTION

The 3DCrane system is a nonlinear electromechanical system having a complex dynamic behavior and creating challenging control problem. It is controlled from PC. Therefore it is delivered with hardware and software which can be easily mounted and installed in a laboratory. You obtain the mechanical unit together with the power supply and interface to the PC and the dedicated digital board configured in the Xilinx technology. The software operates under MS Windows using MATLAB and RTW toolbox package.

The 3DCrane setup (see Fig.1) consists of a payload hanging on a pendulum-like lift-line wound by a motor mounted on a cart. The payload is lifted and lowered in the z direction. Both the rail and the cart are capable of horizontal motion in the x direction. The cart is capable of horizontal motion along the rail in the y direction. Therefore the payload attached to the end of the lift-line can move freely in three dimensions.

The 3DCrane is driven by three DC motors. There are five identical measuring encoders measuring five state variables: the cart coordinates on the horizontal plane, the lift-line length, and two deviation angles of the payload.

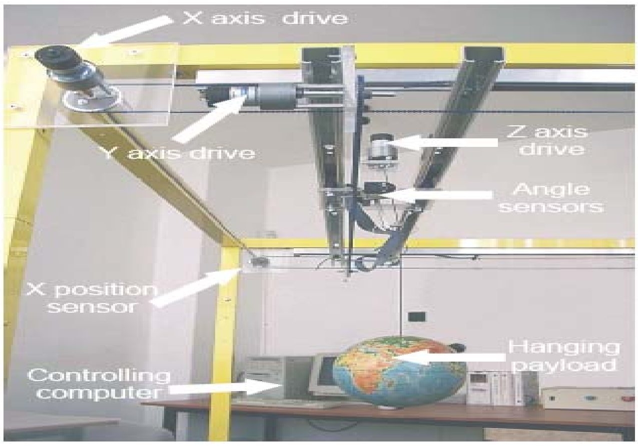

Fig.1. The 3DCrane setup

In the original model, there is no interaction between subsystems, and thus design robust controller for the motion of this 3DCrane system corresponds to design two independent robust controllers for two subsystems. The lift-line R is considered as an uncertainty.

To research the affect of the interaction between subsystems in MIMO system, we consider artificial interactions between the output signals $X(s)$, $Y(s)$ resp. position of card along axes x and axes y. The resulting output signals $X(s)$, $Y(s)$ with interaction corresponding to subsystems are:

$$X(s) = G_{11}(s)\, U_x(s) + \frac{0.25}{8s+1} U_y(s)$$
$$Y(s) = \frac{0.3}{7s+1} U_x(s) + G_{22}(s)\, U_y(s) \quad (1)$$

The main aim of paper is to design robust PID decentralized controllers for the 3DCrane system with interaction (1) for tracking a desired position of the cart on the both case of movement of crane along axes-x and axes-y.

Procedure to design robust PID decentralized controllers can be summarized as five sequential steps:

1. Choose a suitable control configuration, and then identify motion process of the 3DCrane along axes-x and axes-y at three operating points with lift-line z equals 0[m], 0.25[m], 0.5[m] resp. P_1, P_2, P_3
2. Check the selection of the control configuration for this system (Neitherlinski index (NI)).
3. In the case of succeeding selection of the control configuration, create unstructured model uncertainty for motion process. Otherwise, return the first step.
4. Design a robust decentralized PID controller for this process using Small Gain Theorem algorithm and PDLF.
5. Verify obtained result by simulating in Matlab and on the real model.

3. MAIN RESULT

3.1 Identification of positioning process

Using ARX or ARMAX identification method, result of process identification is described by the following transfer function matrices at three operating points.

Transfer function matrix of system at P1 is:

$$G_1(s) = \begin{pmatrix} \frac{-0.04723s+3.363}{s^2+13.64s+0.4507} & \frac{0.25}{8s+1} \\ \frac{0.3}{7s+1} & \frac{-0.01698s+3.032}{s^2+10.52s+0.07569} \end{pmatrix} \quad (2)$$

Transfer function matrix of system at P2 is:

$$G_2(s) = \begin{pmatrix} \frac{-0.03037s+3.163}{s^2+13.03s+0.4558} & \frac{0.25}{8s+1} \\ \frac{0.3}{7s+1} & \frac{-0.05296s+5.292}{s^2+18.82s+0.007261} \end{pmatrix} \quad (3)$$

Transfer function matrix of system at P3 is:

$$G_3(s) = \begin{pmatrix} \frac{-0.01513s+2.345}{s^2+9.741s+0.3612} & \frac{0.25}{8s+1} \\ \frac{0.3}{7s+1} & \frac{-0.01679s+4.165}{s^2+14.71s+0.08042} \end{pmatrix} \quad (4)$$

3.2 Check the selection of control configuration

In this section, we test the given selection of control configuration with nominal model $G_0(s)$ by using Neitherlinski index (NI.).

The Neitherlinski index (NI) is calculated by equation

$$NI = \frac{\det K}{\prod_{i=1}^{2} k_{ii}} \cong 1 > 0 \quad (5)$$

where K is stead-state gain matrix of system.

The positive value of Neitherlinski index indicates that, system is structurally stable.

The given selection of control configuration is correct.

3.3 Design of robust decentralized PID controller using method of equivalent subsystems

Consider the MIMO system described by a transfer matrix function $G(s) \in R^{mxm}$ and a decentralized controller $R(s) \in R^{mxm}$. For robust decentralized control procedure we have used the originally developed method, Method of Equivalent subsystems. For the ESM local controllers are designed according to the independent design approach using any frequency –domain design procedure. Resulting local controllers guarantee fulfillment of performance requirements imposed on the full system. Robust stability and performance is guaranteed using Small Gain Theorem. The design procedure of ESM approach the reader can consult at (Kozáková et al. 2009) and (Osuský et al. 2011).

For the following parameters the robust controller has been designed: demanded phase margin of equivalent subsystems is 70 degree (overshoot is about 10 percent), settle time is about 12sec. Using design procedure given at (Kozáková et al. 2009) and (Osuský et al. 2011) for inverse additive type uncertainty

$$G(s) = (I + l_{ia} G_o(s) \Delta(s))^{-1} G_o(s) \quad (6)$$

and robust stability condition

$$\sigma_M(M) < 1, \forall \omega \quad (7)$$

where

$$M = l_{ia} (I + G_o(s) R(s))^{-1} G_o(s) \quad (8)$$

We have obtained the following robust controller R_i :

$$R_i = \begin{bmatrix} 2.798 + \frac{0.5579}{s} & 0 \\ 0 & 2.653 + \frac{0.5073}{s} \end{bmatrix} \tag{9}$$

Due to (7) the slightly modified robust stability conditions is given at Fig.2

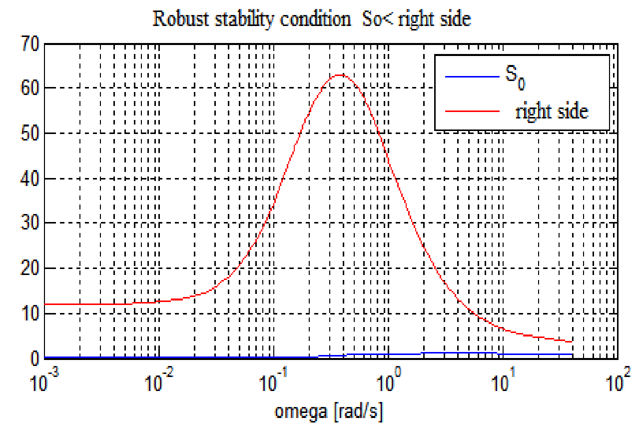

Fig.2.Verifying robust stability condition for input multiplicative model of uncertainty

From Fig.2, we can state that, the closed-loop feedback system with the PI controller R_i is robust stable. And now we verify the obtained result in Matlab simulation (see Fig.3) and on the real model (see Fig.4)

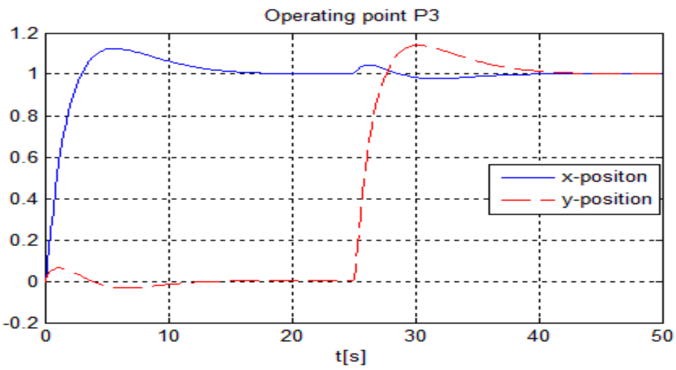

Fig.3. Position output signals with PI controller at third operating point in Matlab

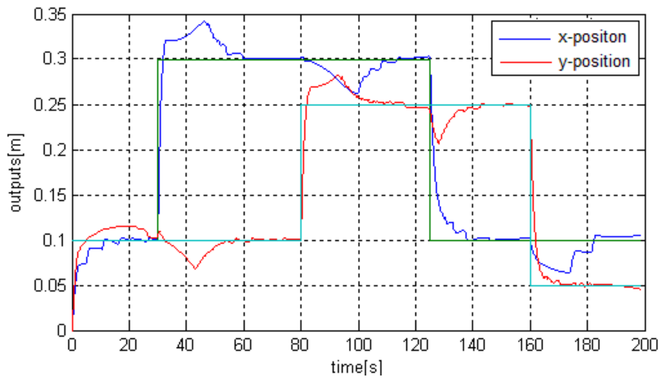

Fig.4 Position output signals with PI controller at third operating point on real model

From results of simulation in Matlab and on the real model we can show that, feedback system with designed PI controllers is robust stable with a demanded performance.

3.4 Design of robust decentralized PID controller using PDLF

Each of the transfer function matrices (2), (3) and (4) can be transformed to linear time invariant continuous system $(A_i, B_i, C, D = 0)$; $i = 1,2,3$, which are considered three vertices of the polytopic system. Our task is to find remain vertex (the fourth vertex) of this polytopic system.

We shall consider the following affine linear time invariant continuous time uncertain system

$$\begin{aligned} \dot{x} &= \left(A + \theta_1 \tilde{A}_1 + \theta_2 \tilde{A}_2\right)x + \left(B + \theta_1 \tilde{B}_1 + \theta_2 \tilde{B}_2\right)u \\ y &= Cx \end{aligned} \tag{10}$$

where $\underline{\theta}_j \leq \theta_j \leq \overline{\theta}_j$; $j = 1,2$;

$$C = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Polytopic model is defined as follow:

$$\begin{aligned} \dot{x} &= A(\xi)x + +B(\xi)u \\ y &= Cx \end{aligned} \tag{11}$$

where

$$A(\xi) = \sum_{i=1}^{4} \xi_i A_i \ , \ B(\xi) = \sum_{i=1}^{4} \xi_i B_i \, , \sum_{i=1}^{4} \xi_i = 1 \ ,$$

$\xi_i \geq 0; i = 1 \ldots N = 4$

Vertices of polytopic system are created by the combination of extreme values of θ_j .

$$\begin{aligned} A_i &= A + \theta_1 \tilde{A}_1 + \theta_2 \tilde{A}_2 \\ B_i &= B + \theta_1 \tilde{B}_1 + \theta_2 \tilde{B}_2 \quad , \quad i = 1 \ldots N = 4 \end{aligned} \tag{12}$$

We suppose that, the extreme values of $\overline{\theta}_j = -\underline{\theta}_j = 1$, $j = 1,2$. Polytopic system will be obtained if for $N! = 24$ combinations of extreme value θ_j , by solving equation system $A_i = A + \theta_1 \tilde{A}_1 + \theta_2 \tilde{A}_2$, $i = 1 \ldots (p+1) = 3$ we have matrices $A, \tilde{A}_1, \tilde{A}_2$ for which the maximal eigenvalues of respective matrix A_4 will be minimal.

The best combination of θ_1, θ_2 for calculation of matrices $A_1, A_2, A_3, A_4 (B_1 \ldots B_4)$ is as follow:

$$\begin{bmatrix} 1 & \underline{\theta}_1 & \underline{\theta}_2 \\ 1 & \overline{\theta}_1 & \overline{\theta}_2 \\ 1 & \overline{\theta}_1 & \underline{\theta}_2 \\ 1 & \underline{\theta}_1 & \overline{\theta}_2 \end{bmatrix} \tag{13}$$

Consider the uncertain system (15), where

$$B = \begin{bmatrix} 3.2630 & -0.0388 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.8333 & 0 & 4.1620 & -0.0350 \end{bmatrix}^T$$

$$\tilde{B}_1 = \begin{bmatrix} -0.5090 & 0.0161 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5665 & 0.0001 \end{bmatrix}^T$$

$$\tilde{B}_2 = \begin{bmatrix} 0.4090 & -0.0076 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5635 & -0.0181 \end{bmatrix}^T$$

$$A = \begin{bmatrix} 0 & -0.4532 & 0 & 0 & 0 & 0 \\ 1 & -13.335 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.333 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0415 \\ 0 & 0 & 0 & 0 & 1 & -14.670 \end{bmatrix}$$

$$\tilde{A}_1 = \begin{bmatrix} 0 & 0.0447 & 0 & 0 & 0 & 0 \\ 0 & 1.9495 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0024 \\ 0 & 0 & 0 & 0 & 1 & -2.0950 \end{bmatrix}$$

$$\tilde{A}_2 = \begin{bmatrix} 0 & -0.0473 & 0 & 0 & 0 & 0 \\ 0 & -1.6445 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0366 \\ 0 & 0 & 0 & 0 & 1 & -2.0550 \end{bmatrix}$$

Consider PID control law as follow:

$$u = Fy + F_d \frac{dy}{dt} = FCx + F_d C_d \dot{x} \tag{14}$$

Closed-loop feedback system with PID controller (19) is:

$$M_d(\xi)\dot{x} = A_c(\xi)x \tag{15}$$

where

$$M_d(\xi) = I - B(\xi)F_d C_d \;,\; A_c(\xi) = A(\xi) + B(\xi)FC$$

Consider cost function as follow

$$J = \int_0^\infty \left(x^T Q x + u^T R u + \dot{x}^T S \dot{x} \right) dt \tag{16}$$

The closed-loop feedback system (15) with the PID controller (14) is robust stable and guarantees the cost function (16) if and only if there exist matrices $P = \sum_{i=1}^{4} \xi_i P_i$, $P_i > 0$; $i = 1 \ldots N = 4$, H, G, F and F_d then the following inequality is satisfied (Rosinová et al. 2007)

$$\begin{bmatrix} A_{ci}^T H^T + H A_{ci} + Q + C^T F^T R F C & \left(P_i - M_{di} H + G^T A_{ci} \right)^T \\ P_i - M_{di} H + G^T A_{ci} & -M_{di}^T G - G^T M_{di} + S \end{bmatrix} < 0 \tag{17}$$

For given cost function with $R = 1 * I$, $Q = S = 0.1 * I$ by using BMI to solve (17), the PID controller is obtained as follow:

$$F = \begin{bmatrix} 1.4065 & 0 & 0.1508 & 0 \\ 0 & 1.1903 & 0 & 0.1218 \end{bmatrix},$$
$$F_d = \begin{bmatrix} 0.1859 & 0 \\ 0 & 0.1579 \end{bmatrix} \tag{18}$$

And now we verify the obtained result in Matlab simulation (see Fig.5) and on the real model (see Fig.6)

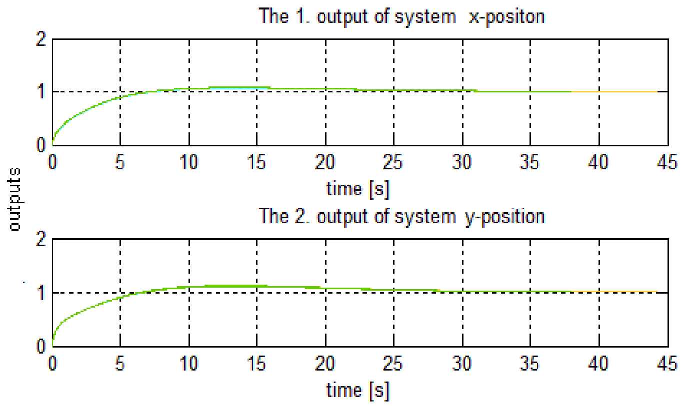

Fig.5. Position output signals with PID controller at three operating points in Matlab

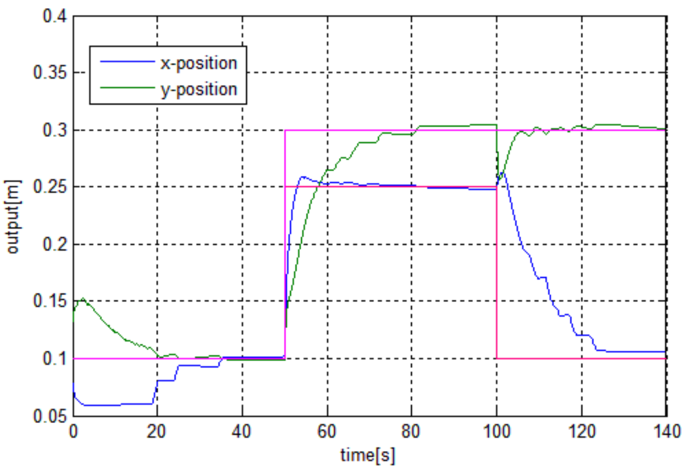

Fig.6. Position output signals with PID controller at third operating point on real model

From simulating results on real model, we can conclude that, the cart of the crane tracks a desired position.

4. CONCLUSION

In this paper, we have researched and applied successfully the knowledge of multivariable system, stabilization of decentralized control systems and the knowledge of robust control theory in the frequency domain and also in the time domain to control the 3DCrane system.

There was a sequential procedure to design robust decentralized controllers for the 3DCrane system. We have identified process of the cart motion along axis x and y. The identification was executed at three operating points corresponding to following length of the payload lift-line: 0, 0.25 and 0.5 [m]. From resulting identified three transfer function matrices, we have designed robust decentralized PID controller to stabilize the cart motion and track the desired position. The resulting PID controllers are verified in the Matlab simulink and on the real model. The cart was at the desired position.

ACKNOWLEDGMENTS

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